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Estimating Displacement Heights and Roughness Lengths over Forests

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Introduction

The wind profile over forests is typically mathematically modelled through a displaced logarithmic wind profile (see [i]).

$$w(z) = v_* \frac{1}{\kappa} \ln\left[\frac{z-d}{z_0}\right] \qquad (1)$$

where *w* is the average wind speed (typically the 10 minute average)
v_* is the friction velocity
z is the height above the ground
d is the displacement height
z_0 is the roughness
κ is the von Kármán constant (~0.40)

This vertical displacement in wind profile is due to the fact that the individual roughness elements (trees) in the forest are situated very closely, so the wind flow is actually lifted to reflect a new surface located at the top of the roughness elements.

Not only is the roughness length within the forest of importance when considering the actual wind profile but also any roughness changes upstream of the forest. Each roughness shift influences the downstream wind profile to some degree.

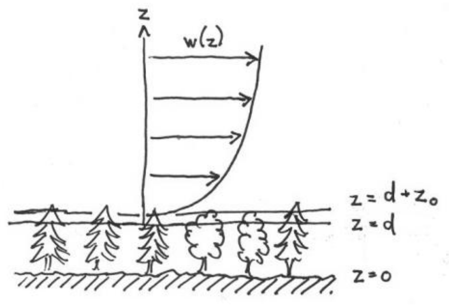

Figure 1: Wind Profile over Forest.

The roughness shifts are modelled using two layers; an internal boundary layer (IBL) and an equilibrium layer (IEL), see the Figure 2. In the lower part of the wind profile – within the equilibrium layer (IEL) – the wind profile is only dependent on the local roughness length, z_{02}. Above the internal boundary layer (IBL), the wind profile is only dependent on the upstream roughness length, z_{01}.

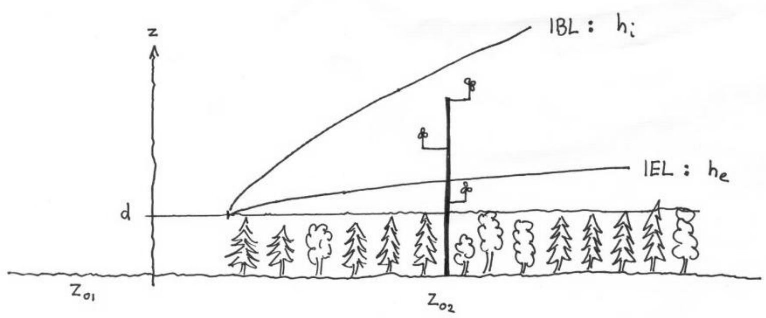

Figure 2: Roughness Shift over a Forest: IBL and IEL.

Between the IBL and the IEL, the wind profile depends on both the z_{01} and the z_{02}. Several models exist for evaluating the heights of the IBL and IEL. However, Dyrbye & Hansen [i] suggest the following simplified equations for evaluating the heights of the IBL (h_i) and the IEL (h_e):

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$$h_i = 0.7 \cdot z_{02} \cdot \left[\frac{x}{z_{02}} \right]^{0.8} \quad (2)$$

$$h_e = h_i \cdot \left[\frac{h_i}{200} \right]^{3} \qquad h_e \le h_i \quad (3)$$

where x is the distance from the roughness shift.

These equations may be used in order to evaluate if a sensor on a meteorological mast is located within or above the IBL or IEL. Please note that other – and more advanced – equations for determining the layers exists, please consult the appropriate literature (e.g. the references [i] or [ii]).

Estimating the Displacement Height and the Roughness Length

Assuming that the meteorological mast is situated deep within a forest, where only the IEL is influencing the wind speed measurements, then it is possible to estimate the displacement height and the roughness length directly from two measurement heights.

Because we are considering the same position, then the friction velocity is constant. Thus we consider both measurement heights (z_1 and z_2,) and then rewrite the equation (1) into the following relation:

$$\frac{w(z_1)}{w(z_2)} - \frac{\ln[(z_1 - d)/z_{02}]}{\ln[(z_2 - d)/z_{02}]} = 0 \quad (4)$$

In an idealized world, then the equation (4) must hold for all measured samples, *i*. However in the real world where the wind profile and wind shear is subjected to influence from stability and local obstacles, then the equation will not hold in general. Instead we choose to determine the displacement height, *d*, and the roughness length, z_{02}, by minimizing the least square sum in equation (5). A similar procedure is reported by Dellwik & Jensen [ii].

Find (z_{02}, *d*) so that $\sum_i \left[\frac{w_i(z_1)}{w_i(z_2)} - \frac{\ln[(z_1 - d)/z_{02}]}{\ln[(z_2 - d)/z_{02}]} \right]^2$ is minimized (5)

The solution to this optimization problem has been implemented in a VBA-code in Microsoft Excel.

Sample Calculations

The VBA-code has been tested on four different meteorological masts situated in dense forests. These forests typically have a characteristic height (h_c) approximately equal to 7.5 meters, which – according to some rules of thumb by Thom [iii] – results in estimates of the roughness length equal to $z_{02} = 0.1 \cdot h_c$ =0.75 m and the displacement height $d = 0.75 \cdot h_c$ = 5.6 m.

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	z_0 [m]	d [m]
Mast 1	1.0	4.5
Mast 2	1.0	6.0
Mast 3	0.7	5.0
Mast 4	1.0	4.5
Rule of thumb	0.75	5.6

Table 1: Results from Sample Calculations

The results from the sample calculations are shown in Table 1. It is seen that the minimization algorithm yields results quite close to the values obtained by the rule of thumb. However, when looking on the optimization maps for the meteorological masts, see Figure ***3*** and Figure *4*, it is seen that surface is very flat near the position of the minima – actually allowing for a broader interpretation of reasonable combinations of (z_0, d). The actual minimum values are marked with an 'X' in the figures below.

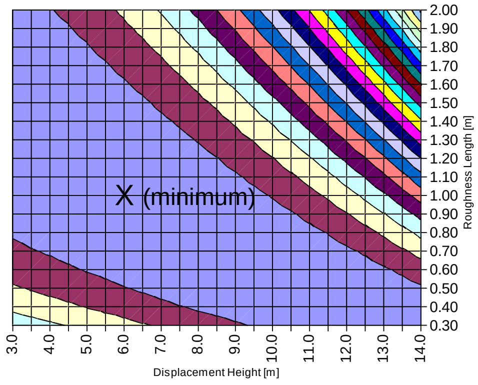

Figure 3: Optimization Map for Mast 2.

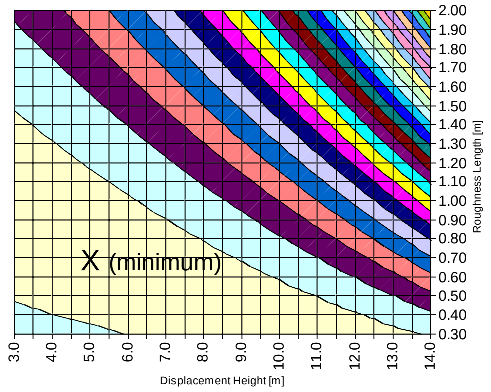

Figure 4: Optimization Map for Mast 3.

Summary

An algorithm has been established in order to provide estimates of the roughness length and displacement heights in shifted wind profiles over forests. The method is implemented in VBA using Microsoft Excel. This method may be applied on sites where the sensors on the meteorological mast are located within the internal equilibrium layer (IEL). Also, the wind speed measurements used to find the (z_0, d) parameters must be reflecting neutral atmospheric stability.

You can download the Excel sheet from the EMD Knowledgebase at the following URL:

http://help.emd.dk/knowledgebase/content/ReferenceManual/EstimateDisplacementHeight.xls

References

[i] Claës Dyrbye & Svend Ole Hansen: *Vindlast på bærende konstruktioner*, SBI 158,1989 (Also available in English: Wind Loads on Structures, John Wiley and Sons Ltd , 1996)
[ii] E. Dellwik & N.O. Jensen: *Internal equilibrium layer growth over forest*, Theory and Applied Climatology, Springer-Verlag, 2000
[iii] A.S. Thom, *Momentum absorption by vegetation*, Quarterly Journal of the Royal Meteorological Society, 97, 414-428