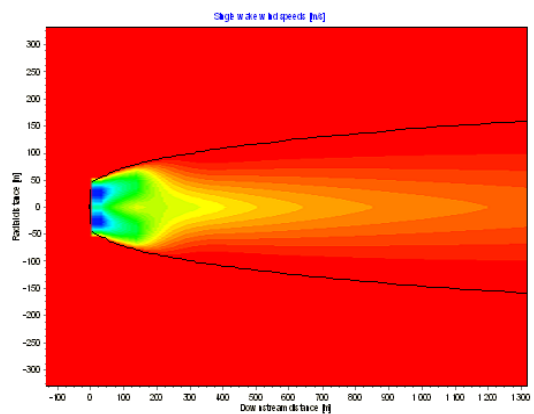
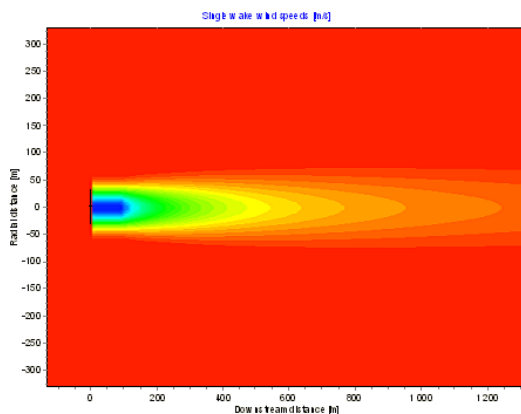
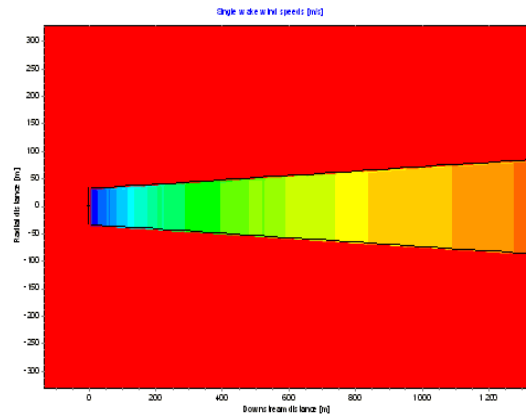


WindPRO / PARK

Introduction to Wind Turbine Wake Modelling and Wake Generated Turbulence



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Front cover

The front cover shows a wake development behind a single turbine. The wake velocities are calculated using the N.O. Jensen PARK model, the Ainslie model (eddy viscosity) and the G.C. Larsen model.

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1. Introduction to Wake Modelling

This paper gives a brief introduction to the concept of wake modelling for arrays of wind turbines. While WindPRO offers the opportunity to calculate the wind farm wakes with a number of different wake models, the user must choose and configure the model that fits the best into the area of application. This report gives selected background theory of single and multiple wake modelling and also on the implementation of the specific wake models. Descriptions of the different wake models are found in the succeeding chapters.



Figure 1: A cluster of four >2 MW semi-offshore turbines in Frederikshavn, Denmark.

Introduction

When the turbine extracts power from the wind, a wake evolves downstream of the turbine. If another nearby turbine is operating within this wake, the power output for this downstream turbine is reduced when comparing to the turbine operating in the free wind. This reduction of power output is – on an annual basis – typically in the range of approximately 2% - 20%, depended on the wind distribution, the wind turbine characteristics and the wind farm (array) geometry.

The turbines operating in the wake are not only subjected to a decreased wind speed but also increased dynamic loading – arising from the increased turbulence induced by the upstream turbines. This increased turbulence must be accounted, when selecting a turbine suitable class of turbines. This is typically done though the specifications in the international codes – e.g. the IEC-61400-1 code for wind turbine structures.

The models available in WindPRO are currently all single wake models, i.e. models capable describing the flow downstream of one turbine. When having multiple turbines, the results from the single wake models are aggregated into a combined result by using empirical combination rules.

The Wake after an Idealized Turbine

Assuming an idealized turbine – where flow around and behind the turbine is without rotation and friction - it is possible to derive some general and important equations describing the wake wind speeds. For further details please consult the publication by Andersen et al. [1]. The derivation is based on the simplified Bernoulli equation, stating that the mechanical energy per unit mass – along a streamline - is conserved:

$$\frac{\rho V^2}{2} + p = H \quad (1)$$

where ρ is the air density
 V is the wind speed
 p is the pressure
 H is the total energy (constant along any streamline)

The Bernoulli equation gives the relation between pressure and wind speed, as the total pressure is constant along a streamline (streamline = a line which is drawn, such as it is always tangent to the velocity vector). Using the Bernoulli equation just before and after the rotor gives us two equations:

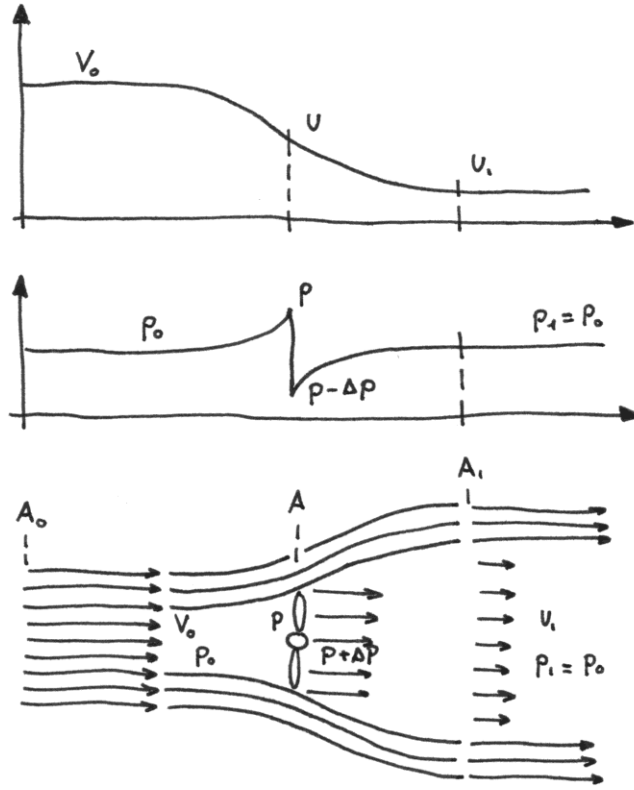


Figure 2: Flow near an idealized turbine: velocity and pressure.

$$\begin{aligned} p_0 + \frac{1}{2}\rho V_0^2 &= p + \frac{1}{2}\rho u^2 \quad \text{and} \\ p - \Delta p + \frac{1}{2}\rho u^2 &= p_0 + \frac{1}{2}\rho u_1^2 \end{aligned} \quad (2)$$

These two equations are then subtracted to yield the drop in pressure over the rotor plane

$$\Delta p = \frac{1}{2}\rho(V_0^2 - u_1^2) \quad (3)$$

Another method for calculating the drop in pressure, Δp , is expressing the drop as the change in momentum of the mass of air passing through one square meter of the rotor area per second (actually by considering the second law of Newton). This yield

$$\Delta p = \rho u(V_0 - u_1) \quad (4)$$

Now equating the equations (3) and (4) gives an expression for the wind speed in the rotor plane:

$$u = \frac{1}{2}(V_0 + u_1) \quad (5)$$

i.e. the velocity in the rotor plane is exactly the average of the far upstream and the far downstream wind speed.

The axial thrust force – i.e. the force acting in the direction of the wind – which is denoted, T , is calculated from knowledge of the pressure difference:

$$T = \Delta p \cdot A \quad (6)$$

where T is the thrust force
 Δp is the difference force
 A is the rotor area

Now defining a ‘axial interference factor’ – a , which is

$$u = (1-a)V_0 \quad \text{and thus} \quad u_1 = (1-2a)V_0 \quad (7)$$

Inserting the equation (7) and equation (3) into the thrust definition equation (6) yields

$$T = 2\rho a(1-a)V_0^2 A \quad (8)$$

Now defining a thrust coefficient, $C_T = 4a(1-a) \Rightarrow a = 1/2 \pm 1/2\sqrt{1-C_T}$, gives

$$T = (1/2) \cdot \rho \cdot V_0^2 A \cdot C_T \quad (9)$$

Inserting the expression for a (and C_T) into the equation (7), this yield

$$u_1/V_0 = (1-2a) = \sqrt{1-C_T} \quad (10)$$

With the equation (10) we now have a relation established between downstream wake wind velocity – u_1 , the turbine thrust coefficient – C_T – and the free wind speed – V_0 . This relation is - using an assumption of the downstream wake expansion – used for making simple and computationally very efficient turbine wake models – like the N.O. Jensen PARK model.

Wake Expansion: When applying the continuity equation in relation with the equation (6) and (10) which are expressions for the wind speed in the rotor plane (u) and far downstream of the turbine (u_1), then an expression for the so-called expanded diameter can be derived [2]:

$$D_{\text{exp}} = RD\sqrt{(1-a)/(1-2a)} \quad (11)$$

Turbulent mixing makes the wind speeds recover to the free wind speeds at some downstream distance, but the equations (10) and (11) can be used to gain insight in the wake expansion rate.

Important Wake Model Parameters

The wake models require different internal wake model parameters as input - as well as a varying number of additional parameters describing the terrain and/or wind climate conditions. Input parameters to a wake model can be turbulence intensity and roughness length. Typically, one would assume that such parameters are depended on the roughness class (or roughness length). In the lack of the preferred measured data, the table below suggests corresponding estimated wake model parameters.

1. Introduction to Wake Modelling

Terrain classification	Roughness Class	Roughness Length	Wake Decay Constant	Ambient Turbulence at 50 m [*] $A_x = 1.8$	Ambient Turbulence at 50 m ^{**} $A_x = 2.5$	Additional detailed description
Offshore. Water areas	0.0	0.0002	0.040	0.06	0.08	Water areas, oceans and large lakes. General water bodies.
Mixed water and land	0.5	0.0024	0.052	0.07	0.10	Mixed water and land. Also applies to the very smooth terrain
Very open farmland	1.0	0.0300	0.063	0.10	0.13	No crossing hedges. Scattered buildings. Smooth hills.
Open farmland	1.5	0.0550	0.075	0.11	0.15	Some buildings. Crossing hedges with 8 m height with distance 1250 m apart.
Mixed farmland.	2.0	0.1000	0.083	0.12	0.16	Some buildings. Crossing hedges 8 m high with distance 800 m apart.
Trees and farmland	2.5	0.2000	0.092	0.13	0.18	Closed appearance. Dense vegetation. 8 m hedges 250 m apart.
Forests and villages	3.0	0.4000	0.100	0.15	0.21	Villages, small towns and much closed farmland. Many high hedges. Forests.
Large towns and cities	3.5	0.8000	0.108	0.17	0.24	Large towns, cities with extended build up areas.
Large build up cities	4.0	1.6000	0.117	0.21	0.29	Large cities with build up areas and high buildings.

* The turbulence intensity is actually calculated based on the assumption of homogeneous terrain with a surface roughness equal to the roughness length. Input to the calculation is also the turbulence measurement height – see the equation below (here based on $A_x = 1.8$, see the equation below).

** Calculated using $A_x = 2.5$, see the equation below.

WindPRO 2.5 assumes that $A_x = 2.5$. Please note that if - during the automated conversions in WindPRO - a terrain classification is exceeding the limits in the table (either the ‘Offshore’ or the ‘Large build up cities’) then the nearest tabular value is chosen.

Estimating the Turbulence Intensity

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain - be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [3, section 3.1.2]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[\frac{1}{\ln[z / z_0]} \right]$$

The value of A is reported to vary approximately between 2.5 to 1.8. κ is the Von Karman constant, which is equal to 0.4. In DS 472 the product between A_x and κ is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC, the characteristic data needed is actually a mean value plus one standard deviation.

Wake Model Limitations – Large Wind Farms and Thrust Coefficient

The wake models are calibrated and tested in small to medium sized wind farms – i.e. wind farms with up to approximately 50-75 turbines. For very large wind farms – 75 turbines or more – the turbines may

influence the surrounding upper air wind climate (Geostrophic winds). In that case, special modelling should be applied – e.g. by ‘artificially’ increasing the roughness within the wind farm.

An important parameter for most models is the thrust coefficient, C_t . The thrust coefficient is used to relate the free wind speed to the downstream wake wind speed through the equation $u_1/V_0=(1-C_t)^{0.5}$. Since the square root is taken, it might be a requirement – depended on the wake model - that the value of C_t is less than 1.0.

Calculating the Annual Energy Production (AEP)

Below is a short description of the algorithm used for calculating the AEP. The wake calculations and the annual energy production (AEP) calculation are actually integrated within the same calculation loop. It is calculated according to the algorithm as outlined in the pseudo code below. Note, that the algorithm assumes that the free wind distribution is based on a modeled Weibull distribution. If measured data is used instead, then the joint distribution table data is used.

```
for iTurbine = 1: N

    Select i Turbine

    Wake Combination for iTurbine calculated (looking at all upstream turbines) /
    Inflow conditions calculated (turbulence and velocity deficits) /
    looping turbines 1 :( iTurbine-1)

    save iTurbine_ThisLoopWakeWSP

    Wake Model for iTurbine:
        - Velocity deficit caused by iTurbine on all downstream turbines
          calculated and stored (from iTurbine+1 to N)

    Incremental AEP calculation (assuming here 12 wind-sectors):
        Find Sector from SectorAngle
        Lookup in weibull table: BinProb=frequency(sector)*AngleStep/30*
        ( F(ThisLoop_WSP)-F(LastLoop_WSP) );
        // F is the cummulative weibull distribution
        AvgPowerInBin:=(Power(iTurbine_ThisLoopWakeWSP)-
                        Power(iTurbine_LastLoopWSP)) / 2;
        AEP_iTurbine:=AEP_iTurbine+AvgPowerInBin*8760*IncProb;

        save iTurbine_LastLoopWakeWSP=iTurbine_ThisLoopWakeWSP

end for
```

Park Power Curves

The park power curves are typically used for different types applications, such as:

1. Forecast systems, based on expected wind conditions at measurement mast. The park power curve is an efficient way to make the connection from wind speed (and direction) to power output.
2. Construction of duration curves, telling how often a given power output will appear. The park power curve can be used together with the average wind distribution at the measurement mast.
3. Calculation of wind energy index based on the wind measurements. The measurements may then be filtered with the park power curve.

4. Checking the actual PARK production versus the expected PARK production based on parallel measurements, e.g. where the expected PARK production is obtained by a lookup in the park power curve table based on measured wind speed and direction.

A part of a typical WindPRO printout from a park-power-curve calculation is found in the Figure 3 below. It is seen, that WindPRO gives the expected power output for each of the sectors and wind-speed-bins. Furthermore two columns are present; the second column below shows the power output for a turbine with no wake effects (column named Free WTGs). Additionally, also a column named 'Park WTGs' are present holding a direction weighted power output. That column would be used in case of analyzing using a meteorological time-series with not directional data present.

Wind speed [m/s]	Power							
	Free WTGs	Park WTGs	N	NNE	ENE	E	ESE	SSE
	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]	[kW]
0.5	0	0	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	0
2.5	0	0	0	0	0	0	0	0
3.5	46	36	18	35	41	41	40	27
4.5	272	236	136	239	258	259	256	193
5.5	542	479	308	484	519	520	515	407
6.5	1,002	876	558	875	950	951	942	735
7.5	1,665	1,475	964	1,479	1,597	1,598	1,585	1,252
8.5	2,411	2,169	1,470	2,187	2,330	2,332	2,316	1,874
9.5	3,204	2,932	2,075	2,972	3,120	3,123	3,107	2,588

Figure 3: Part of a Park-Power Curve Printout from WindPRO.

Calculating the Basic Park Power Curve

The park power curve is much similar to a WTG power curve, meaning that when a given wind speed appears in front of the park with same speed in the entire wind farm area (before influence from the park) the output from the park can be found in the park power curve. Another way to say this: This basic park power curve includes array losses, but do not include terrain given variations in the wind speed over the park area.

The park power curve calculation is implicitly included when doing the a PARK calculation, as the individual turbine results are stored for each sector and each wind speed (the AvgPowerInBin in the pseudo-code above)

Measuring a park power curve is not as simple as measuring a WTG power curve due to the fact that the park power curve depends on the wind direction and that the same wind speed normally will not appear for the entire park area at the same time (only in very flat non-complex terrain). The idea with this version of the park power curve is not to use it for validation based on measurements. This would require at least 2 measurement masts at two sides of the park, unless only a few direction sectors should be tested and also non-complex terrains (normally only useable off shore).

Calculating a Park Power Curve Based on the PPV-model

WindPRO includes a park power curve model that takes into account the influence of the terrain. This model is named WindPRO Park Power Verification model (PPV). The PPV model provides the link between wind data measurements (at a specific measurement position and height) and the calculated production for the wind park. The model includes wake effects as well as the change in wind speed from the measurement position to the position of the WTGs. The influence from roughness, orography and obstacles is modeled based on results from the WAsP model. While the WAsP model is based on statistical data - typically averaged over a year, the link might not represent exactly what is seen when comparing with measurements over a shorter time period. This is due to the yearly variations in stability conditions and other climatic phenomena. The calculation procedure is shown in the pseudo-code below. In essence, the outer looping algorithm is similar to what is being done in a traditional wake loss calculation; however,

an additional inner loop is added in order to model the terrain influences (shown with red in the pseudo-code below).

```
for iTurbine = 1: N

    Select i Turbine

    Wake Combination for iTurbine calculated (looking at all upstream turbines) /
    Inflow conditions calculated (turbulence and velocity deficits) /
    looping turbines 1 :( iTurbine-1)

    save iTurbine_ThisLoopWakeWSP

    Wake Model for iTurbine:
        - Velocity deficit caused by iTurbine on all downstream turbines
          calculated and stored (from iTurbine+1 to N)

    Incremental AEP calculation (assuming here 12 wind-sectors):
        Find Sector from SectorAngle
        Lookup in weibull table: BinProb=frequency(sector)*AngleStep/30*
        ( F(ThisLoop_WSP)-F(LastLoop_WSP) );
        // F is the cummulative weibull distribution
        AvgPowerInBin:=(Power(iTurbine_ThisLoopWakeWSP)-
                        Power(iTurbine_LastLoopWSP)) / 2;
        AEP_iTurbine:=AEP_iTurbine+AvgPowerInBin*8760*IncProb;
        save iTurbine_LastLoopWakeWSP=iTurbine_ThisLoopWakeWSP
        //PPV-Model:
        A: Locate upwind turbine (first turbine with wake influence on iTurbine)
        B: Find mean wind speed at upwind turbine, mast and at iTurbine
        C: Calculate current wind speed at iTurbine using
          terrain induction based on flow results from (B)
        D: Calculate current wind speed at mast using results from (B)
        E: Lookup turbine power yield
        F: Save turbine power yield in current wind speed 'bin' at the mast

end for
```

Please note, that the wind speeds at the mast position are adjusted with terrain induction, but not with any wake influence. If the mast is located close to the wind farm, then you may consider to use a the 'Wind Speeds Inside Wind Farm' report from WindPRO in order to adjust the apparent wind speeds in the table-printout into a wake reduced wind speed.

References

- [1] P.S. Andersen, U. Krabbe, P. Lundsager, H. Petersen, *Basismateriale for Beregning af Propelvindmøller*, Risø-M-2153(rev.), Forsøgsanlæg Risø, Januar 1980.
- [2] J.G. Schepers: ENDOW: *Validation and improvement of ECN's wakemodel*, ECN-C-03-037, March 2003.
- [3] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition, Risø National Laboratory

2. Introduction to the N.O. Jensen Wake Model

The N.O. Jensen wake model is a simple single wake model. The model is documented in the paper ‘A Simple Model for Cluster Efficiency’ by I. Katić et al [1] and is based on the assumption of a linearly expanding wake diameter. This note gives an introduction to the N.O. Jensen wake model and how it is implemented in WindPRO.

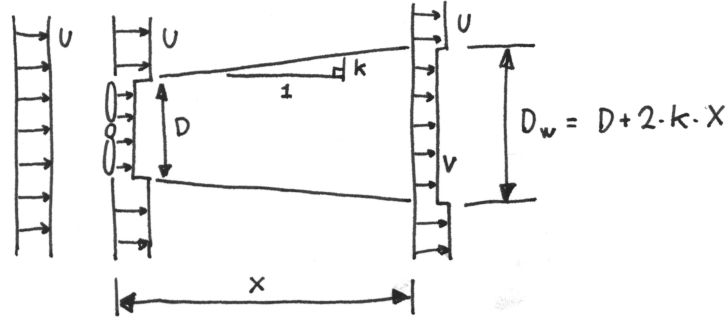


Figure 1: The N.O. Jensen wake model - overview.

Single Wake Calculation

When calculating the velocity deficit, the reduced wind speed, V , downwind of the turbine is derived from:

$$1 - V/U = \left(1 - \sqrt{1 - C_T}\right) / (1 + 2kx/D)^2 \quad (1)$$

Comparing this equation with the previously derived equation for the wind speed just downwind of the turbine (2), it is obvious that the assumption is a linearly expanding wake width.

$$V_0/U = \sqrt{1 - C_T} \quad (2)$$

where

V_0 is the wind speed directly after the turbine of consideration

However it is noticed, that it is not the actual wake wind velocity that is subject for this expansion assumption, but rather the velocity deficit $\delta V_i = (1 - V_i/U)$. Note, that the velocity deficit is defined through the free wind speed, U .

A plot from a calculation in WindPRO is shown in Figure 2. Note, that the wake velocity deficit is uniform given a certain downstream position.

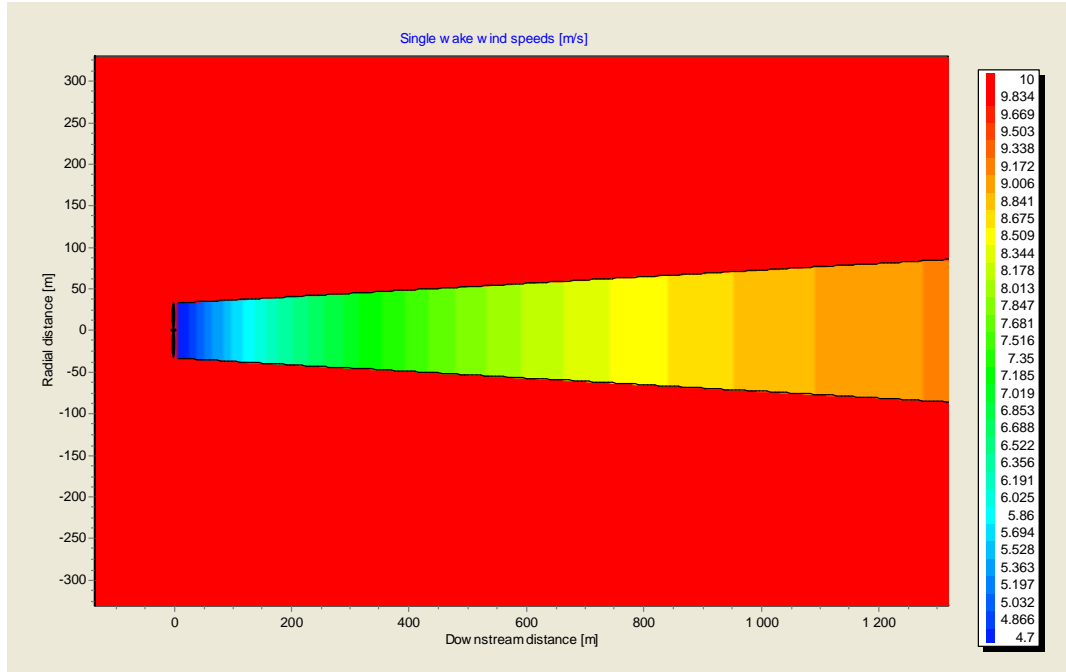


Figure 2: N.O. Jensen wake model - wake development after a single turbine.

Multiple Wake Calculation

Katic et al. [1] suggests, that multiple wakes are calculated through the ‘sum of squares of velocity deficits’ wake combination model. Thus, the N.O. Jensen model initially implemented in the WindPRO PARK module as well as the WAsP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution. The combined effects of multiple wakes are found as:

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2}$$

This model is treated in a succeeding chapter.

References

[1]I. Katić, J. Højstrup & N.O. Jensen, *A Simple Model for Cluster Efficiency*, European Wind Energy Association, Conference and Exhibition, 7-9 October 1986, Rome, Italy.

3. Introduction to the Ainslie Wake Model (Eddy Viscosity Model)

Introduction

The wind turbine wake application of an axi-symmetric formulation of the time averaged Navier Stokes equations with an eddy viscosity closure was initially made by Ainsley [3]. The application uses cylindrical coordinates and an assumption of incompressible fluid. A graphical overview of the model setup is shown in Figure 1.

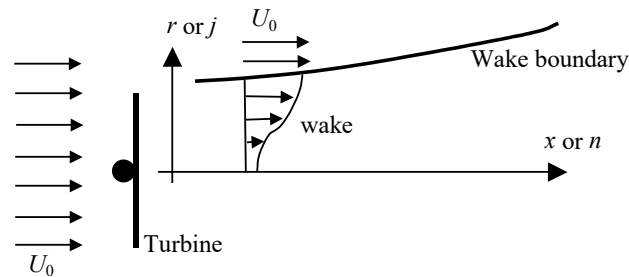


Figure 1: Flow around the turbine.

A result from an implementation of the model is shown in Figure 2, where the wake development behind a turbine with a 50 meters rotor is shown. The calculation is initiated at distance two-rotor diameters downstream (100 meters). Note, that the model calculates the flow through half of the rotor as indicated on Figure 1); this is due to the symmetry assumption used within the model. The free stream velocity is 8.0 m/s (as shown in the legend to the left), while the minimum velocity behind the turbine is 6.5 m/s. The trust coefficient – for this sample calculation – was set to 0.7.

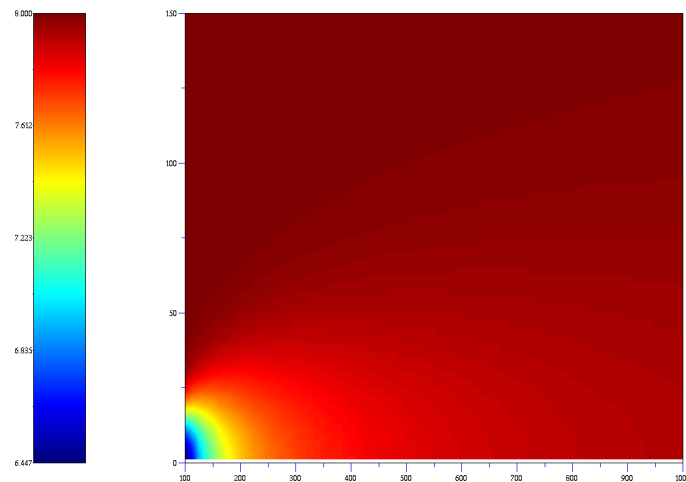


Figure 2: Wake development behind a turbine with a 50-meter rotor.
Note that the calculation is initiated at $2 \cdot RD$ downstream.

Eddy Viscosity Models in windPRO 3.5+

From windPRO 3.5 – released in summer 2021 – a re-implementation of the Eddy Viscosity (EV) model was made – including simplified equations, integration of the maximum wake deficit combination method – and the deep-array-correction (DAC) model. This re-implementation is now our recommended choice for wake modelling using the EV approach – see the Section 3.2 for details on the implementation. The original implementation of the eddy viscosity model in windPRO – available for backward compatibility - is described in section 3.1. In windPRO, the naming convention for these two models are:

- **Eddy Viscosity Model (J.F. Ainslie) : 1988**
Our recommended 2021 implementation with Runge-Kutta solver, DAC and maximum deficit method
- **Eddy Viscosity Model (J.F. Ainslie) : 1986**
2005 implementation with finite difference solver and squared-sum-of-deficits combination method

3.1 Ainslie Wake Model (Eddy Viscosity Model) – 1986

This section 3.1 describes the 2005 implementation of the EV-model in windPRO. This section holds a general introduction to the EV-model and specific features for the 2005 implementation. However, we recommend using the 2021 implementation of the model (see more in section 3.2).

Nomenclature

I_T	turbulence intensity
v	mean (averaged) velocity in radial direction
u	mean (averaged) velocity in axial direction
U_0	mean (averaged) velocity in free flow
σ_u	standard deviation of wind speed process
\overline{uv}	Reynolds stress
$\varepsilon(x)$	eddy viscosity

Continuity Equation (axisymmetric case: $\partial/\partial\phi=0$)

The continuity equation in cylindrical coordinates is (Shames [1]):

$$\frac{1}{r} \frac{\partial rv}{\partial r} + \frac{\partial u}{\partial x} = 0$$

Navier Stokes Equation

In the thin layer approximation and using cylindrical coordinates, the Navier Stokes equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = -\frac{1}{r} \frac{\partial(\overline{ruv})}{\partial r}$$

The last part of the equation above refers to the change in acceleration and thereby momentum. It is not possible to describe this contribution using the velocities in the averaged flow. The part is due to the change in momentum caused by the turbulent fluctuations.

Eddy viscosity (or turbulent exchange coefficient for momentum)

The eddy viscosity is defined in Tennekes and Lumley [2], and is used for establishing an interaction between mean flow and turbulent eddies.

$$-\overline{uv} = \varepsilon(x) \frac{\partial u}{\partial r}$$

According to Ainslie [3], the eddy viscosity, $\varepsilon(x)$, is adequately described by a length scale $L(x)$ and a velocity scale $U(x)$.

$$\varepsilon(x) = L(x) \cdot U(x) + \varepsilon_a$$

The length and velocity scales are taken to be proportional to the wake width b and the velocity difference across the wake shear layer (i.e. independent of r). ε_a is the contribution from ambient turbulence to the eddy viscosity. The length scales are determined by:

$$L(x) \cdot U(x) = k_1 \cdot b \cdot (U_0 - u_0(x))$$

3 case studies showed $k_1 = 0.015$.

Boundary conditions

Ainsley [3] gives the boundary conditions at two rotor diameters downstream of the turbine. The BC at this section is given as a Gaussian velocity profile with the input of initial velocity deficit D_M and wake width b :

$$1 - \frac{U}{U_0} = D_M \exp \left[-3.56 \cdot \left(\frac{r}{b} \right)^2 \right]$$

Empirical data (wind tunnel studies) showed the following equations may be used for determining the velocity deficit and the wake width (A is ambient turbulence intensity in percent):

$$D_M = C_T - 0.05 - (16C_T - 0.5)A / 1000$$

$$b = \frac{3.56C_T}{8D_M(1 - 0.5)D_M}$$

Other authors specify a boundary condition where the initiation position (downwind position) varies. In Lange et al [4] reference to a study made by Vermeulen [5] is made. Vermeulen suggests that the near wake length is modeled through contributions from ambient turbulence, rotor generated turbulence and shear generated turbulence. The near wake length is divided into two regions; the first x_h is modeled as:

$$x_h = r_0 \left[\left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_\lambda^2 + \left(\frac{dr}{dx} \right)_m^2 \right]^{-0.5}$$

where r_0 is an 'effective' radius of an expanded rotor disc, $r_0 = [D/2] \sqrt{(m+1)/2}$ and $m = 1/\sqrt{1-C_t}$

D is the rotor diameter

C_t the thrust coefficient

The different contributions in the equation above are calculated as:

$$\left(\frac{dr}{dx} \right)_a = \begin{cases} 2.5I + 0.05 & \text{for } I \geq 0.02 \\ 5I & \text{for } I < 0.02 \end{cases} \quad \text{ambient turbulence}$$

$$\left(\frac{dr}{dx} \right)_\lambda = 0.012 B \lambda \quad \text{rotor generated turbulence}$$

$$\left(\frac{dr}{dx} \right)_m = \left[(1-m)\sqrt{1.49+m} \right] / (9.76(1+m)) \quad \text{shear-generated turbulence}$$

where I is the ambient turbulence intensity

B is the number of rotor blades

λ is the tip speed ratio

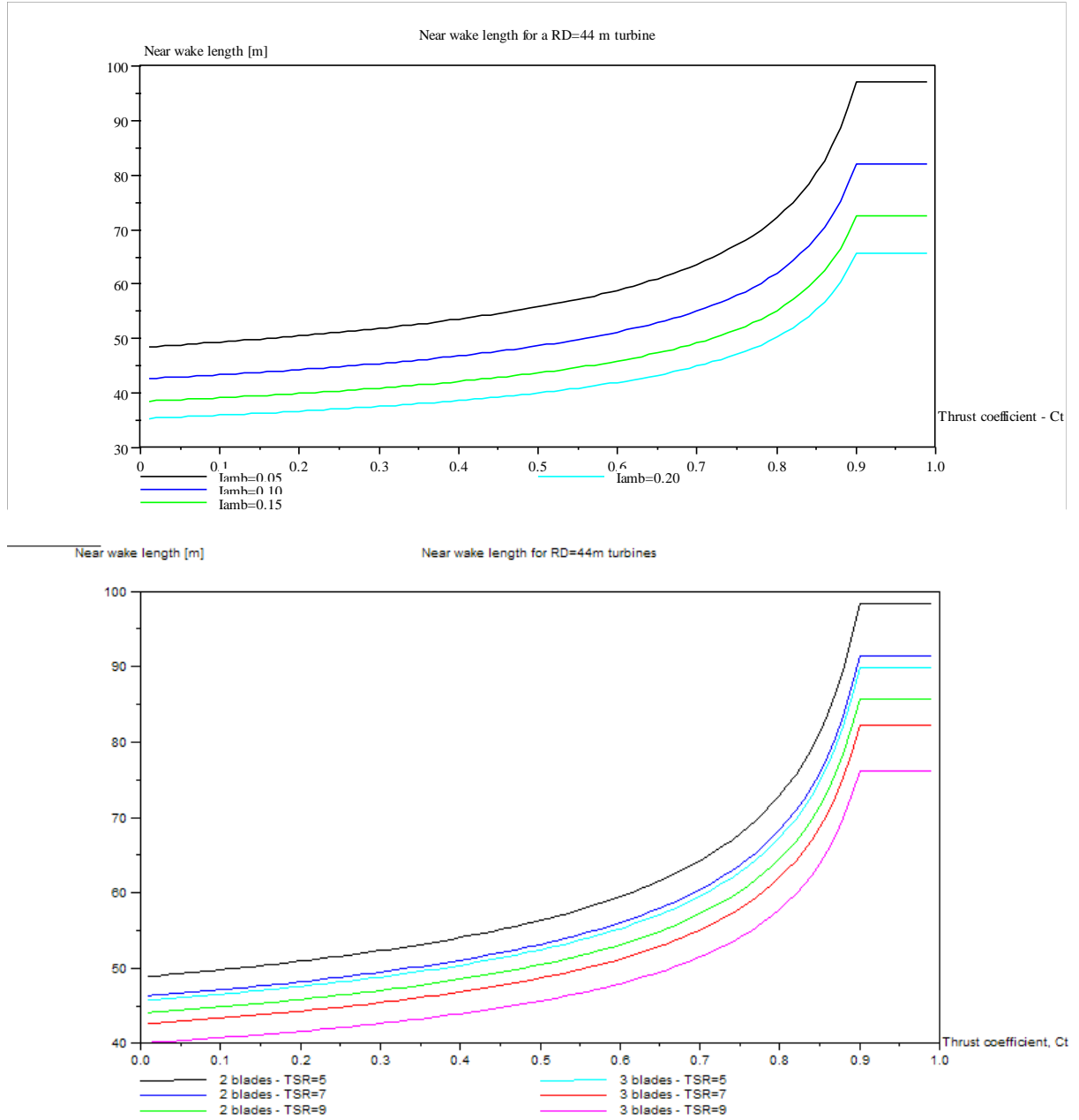


Figure 3: Near Wake Length for a 44 meter Rotor-Diameter Turbine using the Vermeulen Equations. Top: Sensitivity to ambient turbulence. Bottom: Sensitivity to type of turbine and tip speed ratio. Note, that the near wake length is decreasing with increasing ambient turbulence levels.

When the first near wake region, x_h , have been calculated, one can calculate the full near wake length, x_n , by:

$$x_n = \frac{\sqrt{0.212 + 0.145m}}{1 - \sqrt{0.212 + 0.145m}} \frac{1 - \sqrt{0.134 + 0.124m}}{\sqrt{0.134 + 0.124m}} x_h$$

Lange [4] reports that the equations save a singularity at about $C_t=0.97$, so it is suggested that for C_t 's larger than 0.9, then the value for C_t equal to 0.9 is used. A sample calculation for a 44-meter rotor diameter turbine is shown in

Numerical solution method

The differential equation is solved using a finite difference method using a generalized Crank-Nicholson scheme. The solution procedure followed is outlined in Wendt [6]. The numerical solution method used for solving the Navier Stokes equation is made by replaced the differential equation with the finite difference approximations. This approximation introduces truncation errors into the equation.

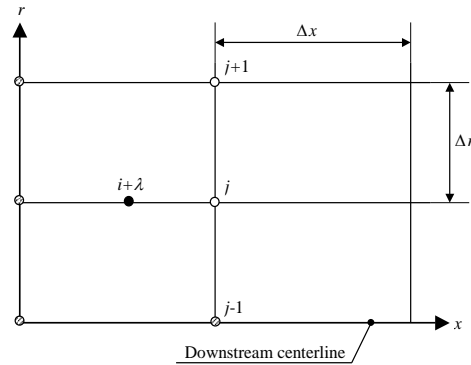


Figure 4: Grid for the generalized implicit method.

Outline of the Solution Procedure

The solution of the partial differential equations invokes an iterative solution procedure. From the boundary condition, the continuity equation is solved. Then the downstream momentum equation is solved in order to get the next downstream velocities. This solution is obtained through an iterative process – the iteration is stopped when convergences is achieved.

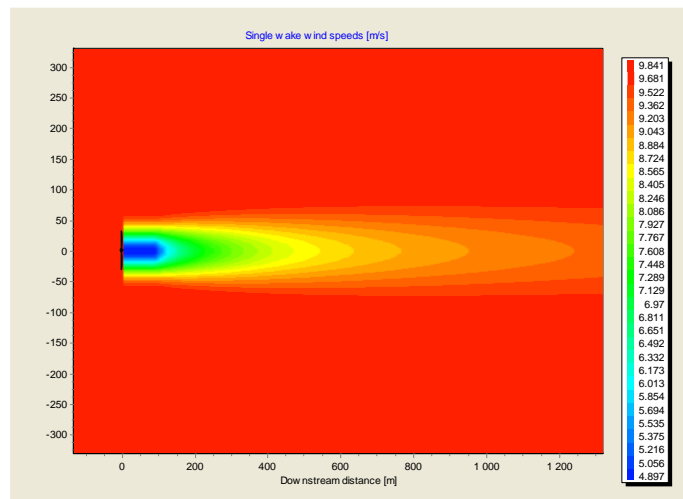


Figure 5: Eddy viscosity model – calculation from WindPRO.

A plot from WindPRO is shown in Figure 5. Note, that wind speeds within the near wake zone are approximated through the solution at the near wake distance.

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3.2 The Ainslie Wake Model with Deep Array Correction Ainslie Wake Model (Eddy Viscosity Model) – 1988

This section 3.2 describes the 2021 implementation of the EV-model available from within windPRO 3.5. This section holds a general introduction to the simplified EV-model equations and specific features for the 2021 implementation – such as the maximum deficit combination model and the DAC model. The 2021 implementation of the Ainslie model is our recommended choice for EV-model in windPRO.

The underlying wake model is the same as described in the Ainslie wake model (Section 3.1) with a few changes. Most notably, the wake combination model only takes the contribution of the most severe wake into account, which works well for single rows of turbines. However, for the larger turbine arrays that have become common since the model was developed, there is a need to account for the deep array effects. Because of that, a Deep Array Correction (DAC) model is introduced. The underlying physics is that in larger arrays the lateral wake expansion and momentum entrainment is limited by the neighboring turbines. The DAC model takes the form of a boundary layer growth model – see more in the following text.

Nomenclature

I_T	turbulence intensity
v	mean (averaged) velocity in radial direction
u	mean (averaged) velocity in axial direction
U_o	mean (averaged) velocity in free flow
\tilde{U}_c	centerline velocity
σ_u	standard deviation of wind speed process
\overline{uv}	Reynold's stress
$\varepsilon(x)$	eddy viscosity
x	downstream distance
r	radial distance from the centerline
d	rotor diameter
b	wake diameter
D_M	velocity deficit

The Ainslie Wake Model

The wake model is similar to the Ainslie wake model in Section 3.1 with a few modifications. The Ainslie wake model is implemented using the simplification suggested by M. Anderson (2009) [2]; it is assumed that the wake will keep its Gaussian shape that it was initialized with. With this simplification the solution reduces the problem from a partial differential equation to a differential equation. The solution is nondimensionalized for simplicity, as seen in Equation 1 below.

$$\tilde{u} = \frac{u}{U_o}, \quad \tilde{U}_c(x) = \frac{U_c(x)}{U_o}, \quad \tilde{r} = \frac{r}{d}, \quad \tilde{x} = \frac{x}{d}, \quad \tilde{\varepsilon} = \frac{\varepsilon}{U_o d} \quad (1)$$

The centerline velocity (\tilde{U}_c) is given as seen in Equation 2.

$$\frac{d\tilde{U}_c}{d\tilde{x}} = 16\tilde{\varepsilon} \frac{\tilde{U}_c^3 - \tilde{U}_c^2 - \tilde{U}_c + 1}{C_T \tilde{U}_c} \quad (2)$$

The formulations seen in Equation 1 and 2 severely simplifies the calculation, and the solution is done using the Runge-Kutta method.

The relation between the nondimensional center velocity (\tilde{U}_C) and the velocity deficit (D_M) can thus be expressed by Equation 3.

$$D_M(x) = 1 - \tilde{U}_C(x) \quad (3)$$

And the flow field is expressed by Equation 4, assuming to follow a Gaussian shape:

$$\tilde{u}(\tilde{x}, \tilde{r}) = 1 - D_M(\tilde{x})e^{-3.56 \tilde{r}^2 / \tilde{b}^2} \quad (4)$$

The wake gets initialized at two rotor diameters downstream as suggested in the original paper by Ainslie [1], instead of calculating the near wake distance as in Section 3.1.

$$D_M(\tilde{x} = 2) = C_T - 0.05 - (16C_T - 0.5)I_0/10 \quad (5)$$

Wake superposition is necessary, as influences in individual turbine wakes are calculated independently. It is assumed that wakes are developing independent from each other solving Equation 2. The resulting wake reduced wind speed at the rotor is calculated by taking the maximum of all velocity deficits and scaling with the local free wind speed:

$$U = U_0 \max(D_{M,j}(r)) \quad (6)$$

where, j is the index for all upstream turbines. Next, follows an introduction the correction applied for large wind farms.

Deep Array Correction for Large Wind Farms

The Deep Array Correction (DAC) model implemented in windPRO is based on the theory presented in [3] and [4] and further the application and implementation are inspired by the work of Schlez et. al. described in [5]. The base assumption for the methodology is, that it builds on the “maximum wake assumptions” for the wake influence. This assumption can be too conservative for large wind farm clusters.

As presented by S.T. Frandsen (2007) [6], a large wind farm is causing its own boundary layer to grow and the wind farm boundary layer change can be described by an increased roughness. Sempreviva et al. (1988) [7] presented a model for describing the flow response to changes in the surface roughness (Equation 5 and 6 within the reference) . When the flow moves over a change in surface roughness an internal boundary layer (IBL) of height h grows. As the ratio of height of the IBL and the scale height approaches zero, the IBL can be expressed as follows:

$$\frac{h}{z_0} \left(\ln \frac{h}{z_0} - 1 \right) = C \left(\frac{x}{z_0} \right) \quad (7)$$

Where,

- x is the downwind distance from the change, m
- z_0 is the roughness length in the wind farm, m
- h is the height of the IBL, m
- C is a constant: $C = 0.9$

The boundary layer growth is sketched in Figure 1. The roughness length in the wind farm is expressed as an added roughness length to the background, as:

$$z_{02} = z_{01} + \Delta z_0.$$

From experimental studies and experiences [5], [8], it is chosen to use a default value of $z_{01} = 0.0002$ and $\Delta z_0 = 0.02$ - 0.03 m for offshore conditions. For onshore conditions we expect to use the background roughness to z_{01} and a value of Δz_0 that is dependent on the roughness class.

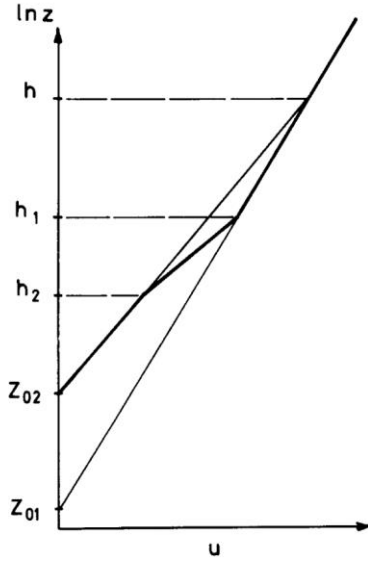


Figure 1: Illustration of the velocity profile in an internal boundary layer for a smooth-to-rough-transition. The outer and inner profiles, illustrates as thin, are matched at $z = h$. The velocity profile is in equilibrium with the surface stress of the IBL until $z = h_2$.

Furthermore, the outer velocity profile reaches down to h_2 ($\approx 1/3 h$) but here used as $0.3h$. And finally, the profile in the range, $h_2 < h < h_1$ has been interpolated [6].

Thus, the wind profile of this new growing boundary layer is expressed in Equation 8.1 - 8.4:

for $z \geq c_1 h$:

$$u(z) = u_{free}(z) \quad (8.1)$$

for $c_2 h < z < c_1 h$:

$$u(z) = u(h_2) + (u(h_1) - u(h_2)) \cdot \frac{\ln(z) - \ln(c_2 h)}{\ln(c_1 h) - \ln(c_2 h)} \quad (8.2)$$

for $z \leq c_2 h$:

$$u(z) = u_{free}(z) \frac{\ln(z/z_{01}) - \ln(z/z_{02})}{\ln(h/z_{02}) - \ln(z/z_{01})} \quad (8.3)$$

Where:

$$u(h_2) = u_{free}(z) \frac{\ln(h_2/z_{01}) - \ln(c_2 h_2/z_{02})}{\ln(h_2/z_{02}) - \ln(c_2 h_2/z_{01})} \quad (8.4)$$

Where,

- c_1 is a constant of 0.3
- c_2 is a constant of 0.09
- h_1 is equal to $c_1 h$, see Figure 1
- h_2 is equal to $c_2 h$, see Figure 1

For this application (large wind farms) momentum is not extracted at ground level and the IBL does not actually starts growing from ground level. Thus, an offset of $2/3$ of the hub height z for the boundary layer height h is applied when calculating $u(z)$ from Equation 8 [2]. Furthermore, it is assumed, that the turbine performance is determined by the ambient wind speed felt at the lower edge of the rotor, i. e. $z = z - \left(\frac{D_{rotor}}{2}\right)$ [2]. To ensure

that the model returns to the free stream a linear zone of returning to the ambient wind speed is introduced. Figure 2 shows an example of the DAC model downstream of a single turbine.

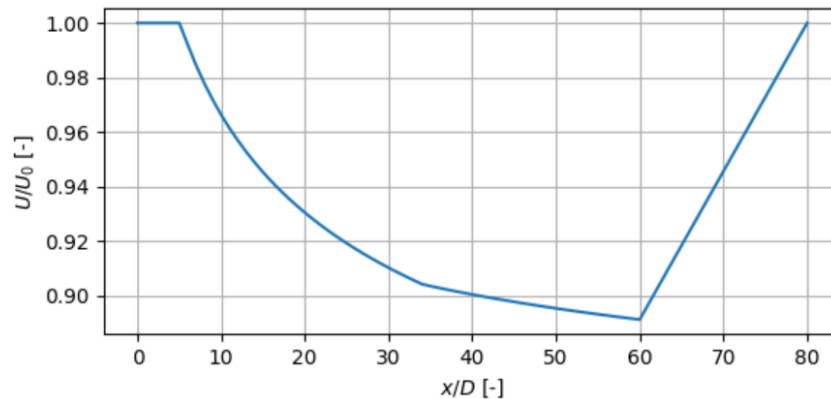


Figure 2: The relative windspeed as a function of the downwind distance given as the numbers of turbine rotor diameters D . In this example, the DAC model kicks in after $5xD$ and after $60xD$ the wind speed starts recovering linearly to the ambient level.

Geometric model

The boundary layer is growing from each turbine downstream with no expansion, following Schlez et. al. (2009) [5]. The deep array correction is therefore only applied to the turbine that are directly downstream of another turbine. Thereby, the deep array correction model is sensitive to the turbine density upstream. An exception is made when wind aligns with the turbine rows and the distance between the centreline of the wakes is at least 5 rotor diameters. For these cases, the wakes of the individual rows are considered to develop independently.

For wind farms with a regular shape the application of the correction is very sensitive to the wind direction. This is detrimental for time varying calculations as small directional differences can lead to large differences in the power output. A Gaussian smoothing is therefore applied that reduces the sensitivity to the direction while retaining the tacit dependency of the DAC model to the wind turbine density.

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4. Introduction to the G.C. Larsen Model (EWTS II)

Introduction

This model is a semi analytical model – derived from asymptotic expressions from Prandtl’s rotational symmetric turbulent boundary layer equations. Because of the asymptotic expressions, the model might be somewhat conservative for close spacings. The model is reported in [1] and is also the recommended wake model – for use with wake loading - in the project report from the European Wind Turbine Standards II Project, an EU-funded project finalized in 1999 [2]. This introduction is based on the EWTS-report [2]. A online introduction of the model can be found in the Risø report ‘A simple wake model’ [3]. Please note that two implementations of the G.C. Larsen model exist within WindPRO. A ‘1999’ version based on the presentation herein (see ref. [2]); and a revised ‘2008’ implementation based on the references [3] and [4].

Model Equations

Assuming that similarity exist between deficits at different downstream positions and only moderate velocity deficits, then the wake radius can be described by:

$$R_w = \left[\frac{35}{2\pi} \right]^{1/5} \left[3c_1^2 \right]^{1/5} [C_T Ax]^{1/3} \quad (1)$$

where

c_1 is a non-dimensional mixing length, described by $c_1 = l(C_T Ax)^{-1/3}$
 l is Prandtl’s mixing length

The c_1 parameter does – according to reference [2] – to some degree separate the rotor drag dependence and thus the c_1 is expected to be relative insensitive to the design and size of the rotor. The equation for determining the c_1 parameter is given in the equation (2) below. Note, that an alternative and approximated specification of the c_1 parameter can be found in section 5.1 of the EWTS II report [2]. The implemented version comes from the reference [4].

$$c_1 = \left[\frac{105}{2\pi} \right]^{-1/2} \left[\sqrt{\frac{1+1/\sqrt{1-C_T}}{2}} \frac{D}{2} \right]^{5/2} [C_T Ax_0]^{-5/6} \quad (2)$$

where C_T is the thrust coefficient

A is the rotor area

D is the diameter of the upstream rotor

x_0 is an approximation parameter, determined by the equation (3) below

$$x_0 = 9.5D \left/ \left(\frac{2R_{9.5}}{D} \right)^3 - 1 \right. \quad (3)$$

In the equation above the $R_{9.5}$ parameter is determined as:

$$\begin{aligned} R_{9.5} &= 0.5[R_{nb} + \min(h, R_{nb})] \\ R_{nb} &= \max(1.08D, 1.08D + 21.7D(I_a - 0.05)) \end{aligned} \quad (4)$$

where I_a is the ambient turbulence intensity at hub height

The wake boundary condition is satisfied, so that the wake radius at the rotor position equals the rotor diameter. Furthermore, empirical boundary condition is applied at 9.5 rotor diameters downstream, where the wake radius is determined from the equation (4). The equation ensures that the minimum turbulence intensity equal to 5% is used, and it essentially states that the wake expansion is dominated by ambient turbulence. The blocking effect of the ground is taken into account by using the design wake radius $R_{9,5}$ in eq. (4) – including the mean of R_{nb} and the minimum of the hub height and R_{nb} .

Mean Wind Velocity Deficit: The mean wind deficit is determined from the expression (5),

$$\Delta V = -\frac{V_a}{9} (C_T A x^{-2})^{1/3} \left\{ r^{3/2} (3c_1^2 C_T A x)^{-1/2} - \left(\frac{35}{2\pi} \right)^{3/10} (3c_1^2)^{-1/5} \right\}^2 \quad (5)$$

where

V_a is the ambient mean wind velocity at hub height

Modified Near Wake Description

The G.C.Larsen wake model includes the option of having a semi-empirical near wake description (second order approach) – enabling the user to model the near wake with a ‘double peak’ velocity profile. This approach is described in detailed in reference [1]. Using the second order option may give a more precise near wake description, especially for densely space turbines. The far wake is not modified.

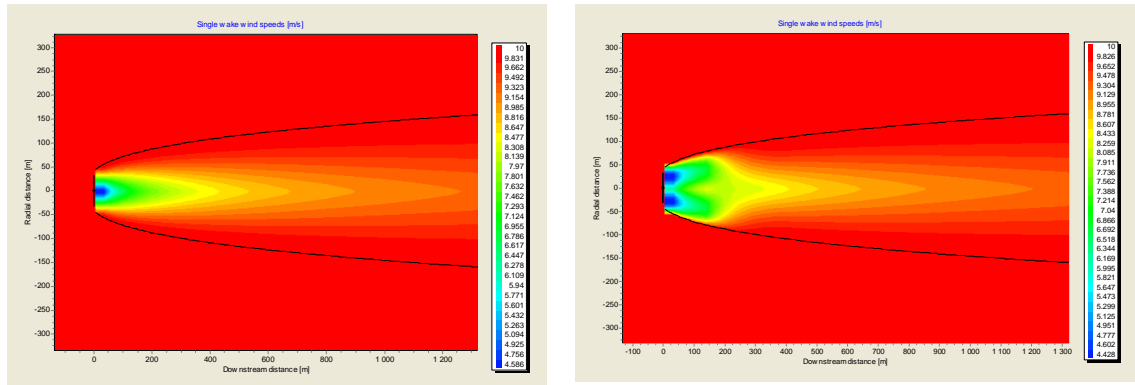


Figure 1: G.C.Larsen wake model (left: First order approach, right: second order approach)

References

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5. Introduction to the TurbOPark Model

Introduction

TurbOPark is a turbulence optimized park model developed by Ørsted R&D [1], [4]. Ørsted R&D released the TurbOPark code – written in MATLAB – as BSD-3 style open source software. In 2023, EMD re-implemented the TurbOPark model based on [3], [4] – so it operates natively from within windPRO 4.0+.

TurbOPark is a version of the PARK model by Katic et al [1] that explicitly uses turbulence intensity (TI) to set the wake decay parameter locally. TurbOPark considers the local turbulence explicitly and does also use wake generated turbulence.

The method was first described by Nygaard et al [2] (2020) and later modified to use a Gaussian wind profile and validated by Ørsted using data from 19 offshore wind farms [4]. The code and parts of the documentation are published and available with a GitHub repository [3].

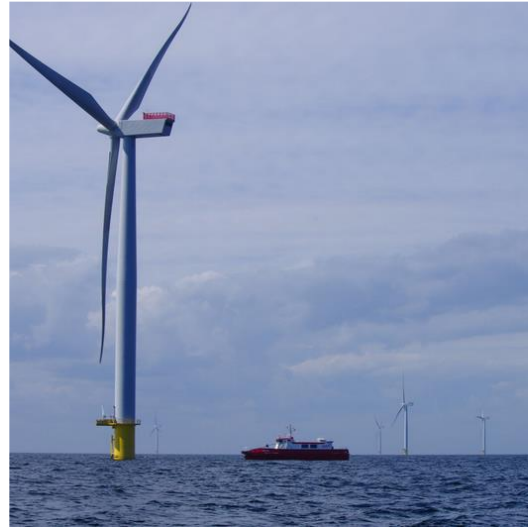


Figure 1:
Service Vessel at Anholt Wind Farm, Denmark.

Documentation, Open-Source Code and Validation

- *Open Source Code:*
Is available from the Ørsted R&D GitHub Account [3]
- *Model Documentation:*
Is available from the Ørsted R&D GitHub Account [3] and the 2020-Torque Conference Paper [2]
- *Validation Paper (19 Offshore Wind farms):*
Is available from the 2022-Torque Conference Paper [4]

Verification of windPRO implementation

We have verified that the implementation in windPRO is the same provided in [3] by recalculating their examples. The example includes a varying background wind speed and a regular grid of 16 turbines of two different turbine types. As an example, the flow case around 6 m/s is shown in Figure 1. The windPRO implementation gives exactly the same results as calculated from the code provided in [3].

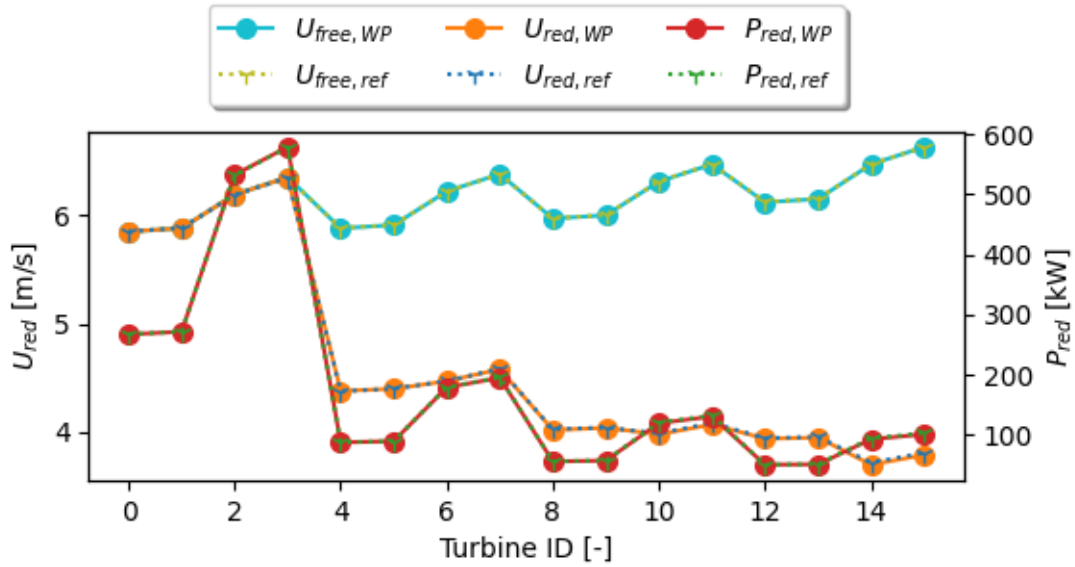


Figure 2: Comparison between windPRO (WP) in solid lines and reference from [3] (ref) in dashed lines for the free wind speed, reduced wind speed and wake reduced power.

References

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6. Wake Combination Models

Most engineering wake models are still single wake models. Thus, in order to obtain a usable result for wind farms with many turbines, these single wakes must be combined into a combined effect. This is done by purely empirical means, using different wake combination models.

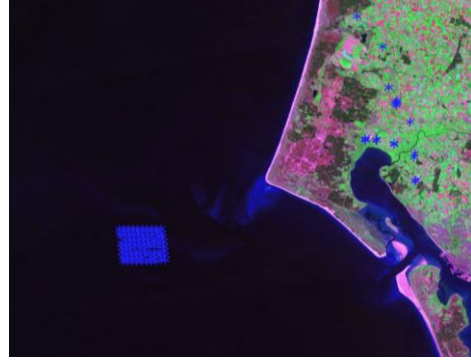


Figure 1: Horns Rev offshore wind farm.

Introduction

Two problems occur when trying to combine the results from a several single wake model into one single downwind wind speed:

1. Since the results from many of the single wake models are non-uniform distributed velocities or velocity deficits, these results must be averaged or combined into an efficient (uniform) wind speed. This is necessary, because the wind turbine power output is to be estimated from the available power curves.
2. When the downwind velocities are determined through one single wake calculation for each turbine, the single wake results must be added into a combined effect.

Ad. 1: Averaging of the Single Wake results

The output from many wake calculations is a non-uniform velocity field. However in order to calculate the power output from a measured power curve, the velocity field must be averaged over the rotor area. In WindPRO, a squared momentum deficit approach is used to calculate this reduction. This approach is similar to the one reported by Lange et. al [1].

$$(u_0 - u_{rotor})^2 = \frac{1}{A} \int_{rotor} (u_0 - u_w)^2 dA \quad (1)$$

where u_0 is the free stream velocity
 u_{rotor} is the averaged velocity at the rotor
 u_w is the non-uniform wake velocity (i.e. a function of the distance and direction from the hub)

Investigations made in connection to the validation of the wake models implemented showed, that using linear combination of wind speeds or using exponents of order 3 only gave marginal differences on the averaged wind speed. The integration in (1) is done by numerical means.

Ad. 2: Wake Combination Models

This averaging may be done in a variety of combinations. Djerf [2] states on option of four different wake combination methods: 1) Sum of squares of velocity deficits, 2) Energy balance, 3) Geometric sum, 4) Linear superposition. According to Djerf it is not recommend using methods (3) and (4). Schepers [3] suggests another approach. Schepers first calculates the wake from the upstream turbine. Then this wake is used for calculating the axial force coefficient on the second turbine downstream. The initial velocity deficit behind the second turbine is then calculated from the axial force.

In WindPRO, the ‘Sum of squares of velocity deficit’ methodology is mainlu used.

Sum of Squares of Velocity Deficits

The N.O. Jensen model initially implemented in the WindPRO Park module as well as the WAsP / Park module uses the sum of squares of velocity deficit to calculate a combined wake contribution.

$$\delta V_n = \sqrt{\sum_{k=1}^{n-1} (\delta V_{kn})^2} \quad (2)$$

where δV is the velocity deficit defined as $(1-V/U)$ – where U is the free wind speed
 n is the number of upstream turbines

Lange et. al [1] uses a slightly different formulation of the sum of the squares of velocity deficits approach. This equation is used in conjunction with equation (1) to calculate the deficit.

$$(u_0 - u_c)^2 = u_0^2 \left(1 - \frac{u_c}{u_0} \right)^2 = \frac{1}{A} \int \sum_{rotor i, all wakes} (u_{rotor(i)} - u_{w(i)})^2 dA \quad (3)$$

Maximum Deficit Method

For the Ainslie Eddy Viscosity model documented in Chapter 3.2, the wake combination model used is the maximum deficit method. For this model, the resulting wake reduced wind speed at the rotor is calculated by taking the maximum of all velocity deficits and scaling with the local free wind speed. For more information, please consult Chapter 3.

Outline of the calculation procedure

An overview of the calculation procedure is as follows:

1. The calculation is initiated with the turbine positioned at the most upstream position (lup turbine)
2. Find (calculate) the wind speed directly upstream of the turbine
3. Calculate the wind speeds downstream of this turbine, i.e. for all downwind turbine positions
4. Calculate the deficits for all downstream turbine positions, i.e. relating to the free wind speed
5. If the downstream turbine is in a partial wake, then reduce the velocity deficit with the fraction of the overlap area to the rotor area of the downstream turbine.
6. Calculate the square of the velocity deficits
7. Continue with the next turbine (using step 1), by summing the squares of the velocity deficits.

References

- [1] Lange, Bernard et al: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy 2003; 6: 87-104
- [2] Djerf, E. & Mattson, H.: *Evaluation of the Software program WindFarm and Comparisons with Measured Data from Alsvik*, FFA TN-2000-30,
http://www.vindenergi.foi.se/Rapporter/TN2000_30_WindFarm.pdf
- [3] Schepers, *ENDOW: Validation and Improvement of ECN's wake model*, ECN-C-03-034.

7. Introduction to Turbulence and Wakes

Wind turbines operating in wakes are subjected to higher turbulence levels than turbines operating in the free wind, thus appropriate turbulence calculations should be made before selecting the proper turbine design class when having clusters of turbines. This is due to the fact that the fatigue loads and possibly also the extreme loads are higher when the turbulence levels increases.

The wake added turbulence may be calculated using different wake or turbulence models. These models are typically very different in detailing level – and possibly also in accuracy. The models range is from simple engineering models to the more advanced computational fluid dynamic (CFD) models. The CFD-models are typically also very demanding in terms of calculation time.



This chapter gives an introduction to the calculations and operations performed on the measured ambient turbulence data, and how the turbulence data from single wake models is merged. We also give a brief introduction to the turbulence calculation required by the IEC 61400-1 structural code.

Figure 1: Part of a Flash Animation Created using WindPRO.

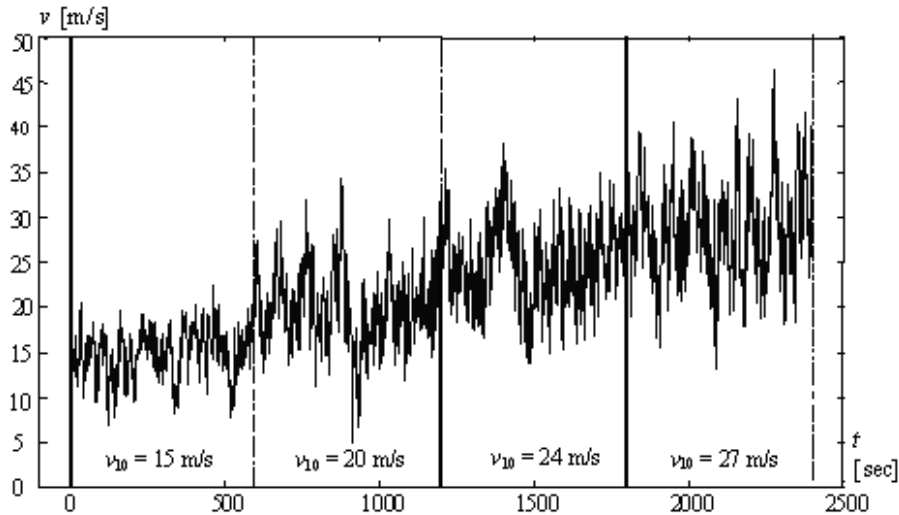


Figure 2: Turbulent winds – simulated at different mean wind speeds, from [1].

WindPRO contains several models for calculating the wake added turbulence. The ambient turbulence level must be user specified, e.g. through measured data or alternatively through the roughness classification.

The Turbulence Calculation

When estimating the design, lifetime and fatigue on wind turbines, the turbulence levels are of outmost importance. The turbulent winds arise from several sources:

1. Orography induced turbulence, i.e. flow over hills and mountains
2. Roughness induced turbulence, i.e. flow generated by objects within the landscape
3. Turbine generated turbulence, i.e. turbulence in the wake of the turbines
4. Obstacle induced turbulence, i.e. turbulence generated in the wake of large nearby obstacles

The turbulence intensity is defined as the ratio between the standard deviation of the wind speed, σ_u , and the 10-minute mean wind speed, U_{10} . When dealing with wind turbine wakes, it is tradition to relate the 10-minute mean wind speed to the free wind speed, i.e. the wind speed outside the wake.

$$I_T = \frac{\sigma_u}{U_{10}} \quad (1)$$

The current edition of WindPRO – version 2.5 – deals primarily with the turbine generated turbulence. Orography and roughness generated turbulences are included only through the on-site meteorological measurements – or alternatively through user-defined turbulence input levels.

Estimating the Ambient Turbulence Level

When doing the wake calculations then the ambient turbulence level must be estimated either through on-site meteorological measurements, through simple roughness classifications or using numerical flow models. Even if the definition of the ambient turbulence seems reasonable simple (see equation 1), then the estimation of this turbulence intensity is quite difficult due to the stochastic nature of the turbulence, i.e. for a given wind speed then measurements of turbulence intensity will show significant scatter. This scatter can be modelled only accurately as a random variable; so as a minimum requirement it is recommended not only to calculate the mean turbulence level, but also the standard deviation of the turbulence intensity. Actually, this is done automatically when you load meteorological measurements in a meteo-object in WindPRO.

In WindPRO three different measures of the ambient turbulence is used and calculated in each bin (wind speed and sector):

1. Mean (average) turbulence
2. Standard deviation of turbulence
3. Representative (characteristic) turbulence

The first two measures are purely statistical estimators, used in order to describe the turbulence distribution. The last issue (3) is included as the structural codes typically require that a design value of turbulence is used; i.e. the representative turbulence is some function of the mean and standard deviation of the turbulence. Actually, this definition of the representative ambient turbulence levels varies also with different structural codes, e.g. the IEC 61400-1 second and third editions [2, 3] have different definitions of this parameter, see below.

Ambient Turbulence Level According to the IEC 61400-1 second edition

When estimating the wind condition to check if an IEC class turbine is suitable for a particular site, then the IEC 61400-1 second editions calls for calculating an I_{15} parameter which is a characteristic value of hub height turbulence intensity at 10 min average wind speed of 15 m/s. The characteristic value is calculated by adding the measured standard deviation of the turbulence intensity to the measured or estimated mean value (only considering the 15 m/s bin values) , i.e.

$$I_{15} = \mu_{I|15m/s} + 1.0 * \sigma_{I|15m/s} \quad (2)$$

The IEC 61400-1 ed. 2 requires the I_{15} parameter to be estimated using statistical techniques applied to wind speeds and turbulence measurements above 10 m/s. It specifies also, that the influence of the wakes should be accounted for.

Ambient Turbulence Level According to the IEC 61400-1 third edition

IEC 61400-1 ed. 3 has a slightly different approach to turbulence modelling as it focuses on the standard deviation, σ_u , rather than the turbulence intensity. The IEC 61400-1 ed. 3 requirement is that the following equation is fulfilled for all wind speeds from 0.6 times the rated wind speed to the cut-out wind speed:

$$\sigma_1 \geq I_{eff} \cdot V_{hub} + 1.28 \cdot \hat{\sigma}_\sigma \quad (3)$$

where σ_1 is the turbulence standard deviation from the normal turbulence model as specified in the IEC code
 I_{eff} is the total turbulence (ambient and wake)
 V_{hub} is the wind speed at hub height level
 σ_σ is the measured standard deviation of the turbulence standard deviation

The factor 1.28 is applied because a 90% percentile is sought.

Calculating the Ambient Turbulence from Measurements

When on site measurements are available then WindPRO is able to calculate the mean turbulence intensity table as well as the standard deviation of the turbulence intensity and a user defined representative turbulence level.

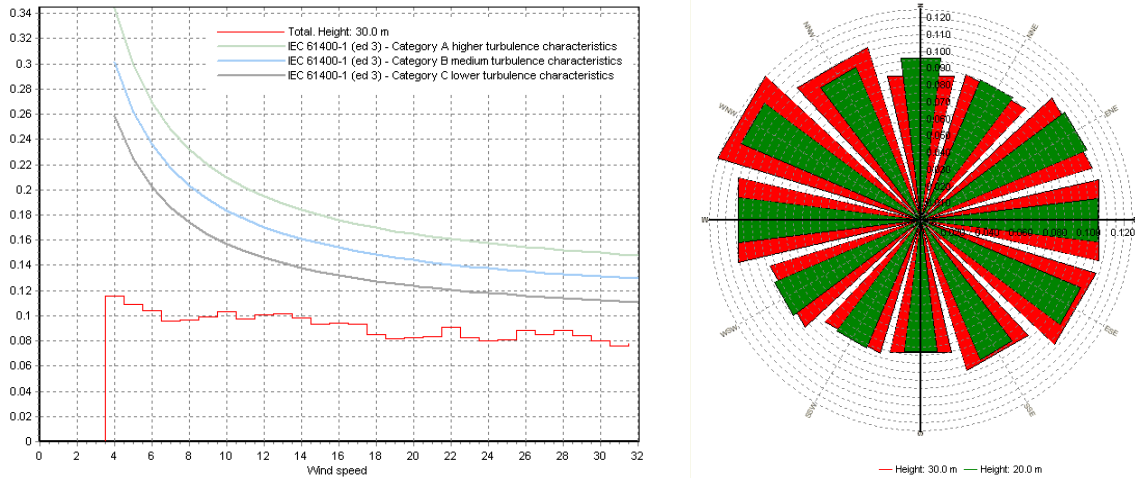


Figure 3: Measured Ambient Turbulence Levels.

Calculating Turbulence Intensity from Roughness Data and/or Roughness Maps

The turbulence intensity on a specific site can be estimated from the roughness rose or directly (in a more raw manner) from the surface roughness in the considered point. The relation between the turbulence and the surface roughness can – in the case of homogeneous terrain - be derived from boundary layer theory to, see Guidelines for the Design of Wind Turbines [4]:

$$E[\sigma_u] = U_{10} A_x \kappa [1 / \ln[z / z_0]] \Leftrightarrow I_T = \frac{E[\sigma_u]}{U_{10}} = A_x \kappa \left[\frac{1}{\ln[z / z_0]} \right] \quad (4)$$

The value of A is reported to vary approximately between 2.5 to 1.8. κ is the Von Karman constant, which is equal to 0.4. In DS 472 [5] the product between A_x and κ is (conservatively) set to 1.0. The estimated turbulence levels from the equation above give a mean level of turbulence. However in relation to IEC 61400-2, the characteristic data needed is actually a mean value plus one standard deviation, so some estimate of the standard deviation of the turbulence is needed.

A Rule of Thumb to Estimate the Standard Deviation of the Turbulence

This standard deviation may be estimated – from a rule of thumb widely used in Germany. In the general case, the coefficient of variation ($COV = \sigma / \mu$) is set to 20%. Only for forest sites and for extreme hill tops, this value is not sufficient but must be replaced by measurements.

Vertical Scaling of the Ambient Turbulence Level

Often, when turbulence measurements are available from the site, the measurements are not taken at hub-height level. This calls for a vertical scaling of the ambient turbulence, which is done by assuming homogeneous terrain (an approximation to the real nature). Preferably, the turbulence should be taken from hub-height measurements.

Assuming that the wind flow is a horizontally homogeneous (i.e. the properties of the flow do not change in the horizontal direction), then the standard deviation of the wind speed process is only depended of the height above the terrain, z .

The turbulence intensity in the height x meters is defined as:

$$IT(x) = \frac{\sigma_U(x)}{U_{10}(x)} \quad (5)$$

Where

IT is the turbulence intensity

σ_U is the standard deviation of the wind speed

U_{10} is the mean wind speed averaged over 10 minutes

Experimental data has shown that the standard deviation of the wind speed only decreases very slowly. In Armit [6] & Dyrbye & Hansen [7], it is said, that it is reasonable to use constant standard deviations up to about the half-height of the internal boundary layer. This assumption is also used in WAsP and in most structural codes.

Using this assumption, the vertical scaling of turbulence intensity between two heights is simply calculated by assuming the same standard deviations in the two heights (x and y meters or feet).

$$\begin{aligned} \sigma_U(x) &= \sigma_U(y) && \Leftrightarrow \\ IT(x) \cdot U_{10}(x) &= IT(y) \cdot U_{10}(y) && \Leftrightarrow \\ IT(y) &= \frac{U_{10}(x)}{U_{10}(y)} IT(x) \end{aligned} \quad (6)$$

So now the problem is reduced into calculating the mean wind speed in the new height. The vertical scaling of wind speeds may be done using the power law vertical wind profile a purely empirical equation. The power law wind profiles also require quite homogenous terrain.

$$U_{10}(y) = U_{10}(x) \cdot \left[\frac{y}{x} \right]^\gamma \quad (7)$$

where

γ is the wind gradient exponent

The wind gradient exponent is known to be very depended on the roughness length or the roughness class. The table below gives guidelines for selecting the wind gradient exponent – if no measured data is available:

Roughness Class	Roughness Length	Wind Gradient Exponent
0	0.0002	0.1
1	0.03	0.15
2	0.1	0.2
3	0.4	0.3

Inserting the equation (4) into (3) we obtain the turbulence scaling law, valid for homogeneous terrain:

$$IT(y) = \frac{U_{10}(x)}{U_{10}(y)} IT(x) = IT(x) \cdot \left[\frac{y}{x} \right]^{-\gamma} \quad (8)$$

Turbulence from Wind Turbine Wakes

The wake added turbulence is either derived from the (single) wake models that include turbulence modelling or from dedicated (empirical) turbulence models. The turbulence calculated from the different models may be parameterized in numerous ways, see e.g. Figure 4 which holds output from the eddy Viscosity wake model. Using the EV-model one may relate the eddy viscosity to the turbulence intensity or alternative use empirical values. Again, other models have wake turbulence included in a purely empirical manner. The turbulence model must be used in connection with a wake model – in order to take the reduced wind speeds in the wind farm into account.

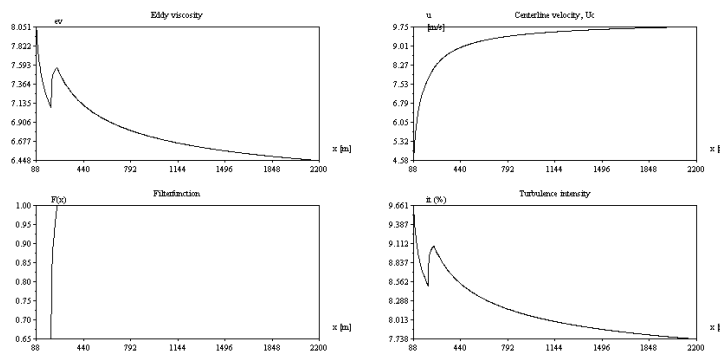


Figure 4: Single Wake Turbulence Modelling using the Eddy Viscosity Model.

The results from the turbulence models – may typically come within one of four categories:

1. Added turbulence model – calculated for the wake after a single turbulence
2. Added turbulence model – calculated for all surrounding turbines
3. Total turbulence model - calculated for the wake after a single turbulence
4. Total turbulence model – calculated for all surrounding turbines

Models (1) and (2) give the wake added turbulence contribution. This should be added to the ambient turbulence level. The model type (3) gives the total turbulence level for a given wake at a given position (ambient and wake added), and this must be summed into a combined effect considering all upstream turbines. The model type (4) gives the total turbulence level in an integrated manner, thus no single wake adding is needed.

All of the turbulence models implemented in WindPRO belong to any of these four types.

Calculating the wake added turbulence intensity

The turbulence intensity is defined as the ratio of standard deviation to the mean wind speed. It is common practice to relate the turbulence intensity – also within the wake – to the ambient free wind speed. Also, it is practice to assume that the added turbulence level may be added as independent stochastic variables.

In the Danish Recommendation [6] the total turbulence intensity is actually calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (9)$$

Partial Wakes - Turbulence

When the turbine operates in a partial wake, we use the equation (6) to calculate the added turbulence level – considering the rotor area with ambient turbulence only. A linear weighting with rotor areas is assumed.

Converting From Time Series Turbulence to Turbulence Tables

From WindPRO 2.5, the meteo object is the container for three different turbulence tables: The representative or characteristic turbulence table, the mean turbulence table and the standard deviation turbulence table. Each of these tables is used to store the turbulence intensities.

Each table with turbulence intensities is typically binned with an angular interval equal to 30 degrees and a wind speed interval equal to 1.0 m/s. In each bin the sample statistics are then calculated (mean and standard deviation), see also [8]:

$$\overline{X} = \frac{X_1 + X_2 + \dots + X_n}{n} \quad (10)$$

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \overline{X})^2}{n-1}} \quad (11)$$

where \overline{X} is the sample mean
 S is the sample standard deviation

These two sample statistics are stored in the mean and standard deviation tables respectively. The representative turbulence table values is calculated using the IEC code relations (or user defined relations) as indicated earlier in this chapter. Not only are the binned statistics stored and presented but actually also omni-directional statistics and the sector-wise results. All of these statistics are – as a default setting – derived directly from the time series data.

Manual Editing of the Mean and Standard Deviation Turbulence Tables

When you choose to manually edit the turbulence tables (mean + standard deviation tables), then the omni-directional and sector-wise means are calculated using the assumption that the binned sample distributions are independent Gaussian distributions.

By using this model to calculate statistics you will typically have results that is only differing a few per mille when compared to statistics based on the measured time series data. This is due to the fact that the samples may not fit perfectly to the Gaussian distribution.

Note: Only the mean + standard deviation tables may be edited. The representative turbulence is calculated automatically based on this table.

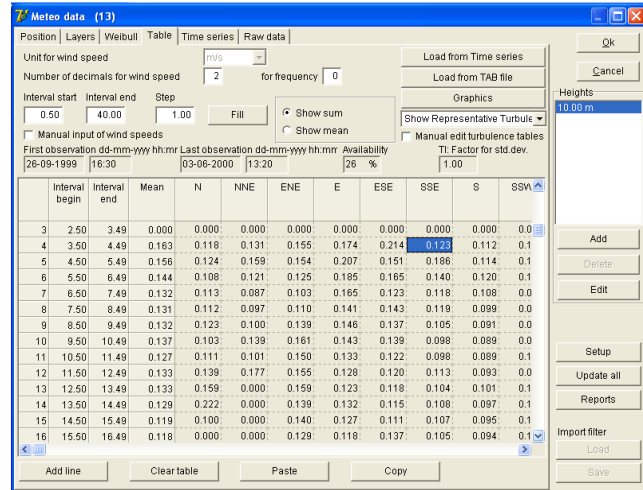


Figure 5: Screen Shot of Meteo Object Turbulence Table in WindPRO 2.5.

If you wish to reset your manual edits, then you must reload tables from the time series data.

We calculate the statistics using a Monte Carlo simulation approach. This approach requires both lookup in the frequency table (to get the number of actual samples in each bin) and lookup in the binned turbulence sample statistics (mean and standard deviation). The approach follows the following procedure, as outlined in this pseudo code algorithm:

```

for each sector-bin (typically 1 - 12) do
  Get mean and standard deviation of selected sector-bin
  Make Gaussian distribution using the mean and standard deviation
  Lookup in frequency table to find number of occurrences (cnt) in this bin
  If wind speed is less than the 'include turbulence wind speed' then cnt = 1
  If cnt = 0 then we assume that cnt = 1
  Use Gaussian distribution to simulate 'count' new occurrences
  Update omni-directional Statistics using the simulated data
end

```

Calculate the omni-directional mean and standard deviations

Wind Speed Interval [m/s]		Omni-directional Turbulence Intensity		
From	To	Time series	Gaussian model	Difference
6.5	7.5	0.132	0.132	0.000
7.5	8.5	0.131	0.132	0.001
8.5	9.5	0.132	0.132	0.000
9.5	10.5	0.137	0.137	0.000
10.5	11.5	0.127	0.127	0.000
11.5	12.5	0.133	0.133	0.000
20.50	21.50	0.099	0.099	0.000
21.50	22.50	0.114	0.114	0.000
22.50	23.50	0.097	0.098	0.001
23.50	24.50	0.093	0.093	0.001
24.50	25.50	0.092	0.097	0.005
25.50	26.50	0.099	0.097	0.002

Table 1: Comparing Selected Results from Turbulence Table Calculations.

A sample calculation – where the representative turbulence has been extracted using a factor on the standard deviation equal to 1.00 - is shown in the Table 1. Please note, that the difference between representative turbulence calculated using the time series data and the Gaussian model data increases when the frequency decreases (typically at very rare bins in the upper tail of the distribution).

References

- [1] Misfeldt & Thøgersen, *Reliability Analysis of Wood Structures*, Aalborg University, 1997 (unpublished)
- [2] IEC 61400-1:1998 (2nd edition)
- [3] IEC 61400-1 (3rd edition)
- [4] DNV & Risø: *Guidelines for the Design of Wind Turbines*, Risø National Laboratory
- [5] DS472, Danish Code of Standards for Wind Turbine Structures
- [6] Armit, *Wind Structures*, Lecture Series, Von Karman Institute for Fluid Dynamics, 1976.
- [7] Dyrbye & Hansen, *Wind Loads on Structures*, John Wiley and Sons, 1996
- [8] Sheldon M. Ross: *Introduction to Probability and Statistics for Engineers and Scientists*, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons, 1987

8. Danish Recommendation – Turbulence Model

The Danish Recommendation [1] from 1992 specifies a quite simple wake added turbulence model. If the turbines are erected in a cluster with a minimum distance between the turbines of 5 times the rotor diameter – or in a row with the distance 3 times the rotor diameter – then a added turbulence intensity of $I_{\text{park}} = 0.15$ can be used. An alternative is to use a mean-contribution, which varies by the mean wind speed and the distance between the turbines:

$$I_{\text{park}} = \beta_v \cdot \beta_l \cdot 0.15 \quad (1)$$

where β_v is a parameter taking the mean wind speed into account (see the Figure 1)

β_l is a parameter taking the distance between the turbines into account (Figure 2 and 3).

The β_l parameters are dependent on the geometrical configuration of the wind farm, i.e. if the wind farm is erected in a row (Figure 2) or in a cluster (Figure 3).

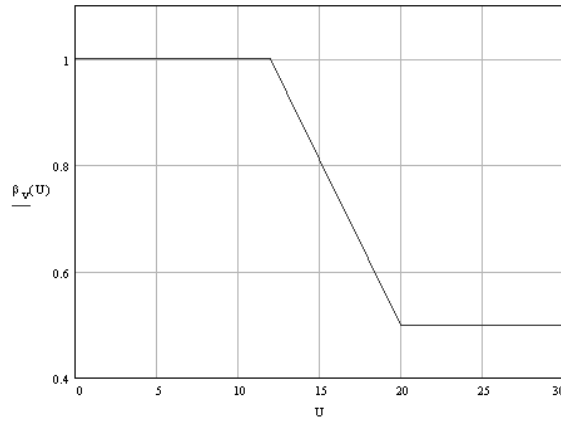


Figure 1: Factor taking wind velocity into account, β_v .

The β_l factor is determined from the Figures 2 and 3.

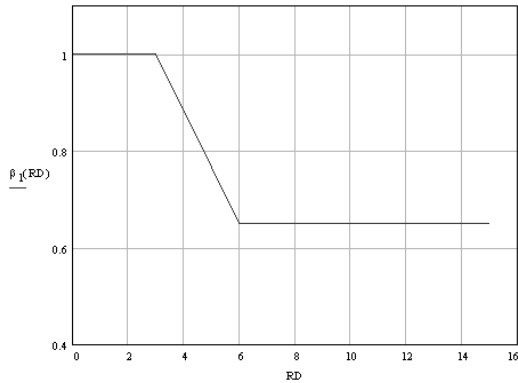


Figure 2: β_l for turbines in a row.

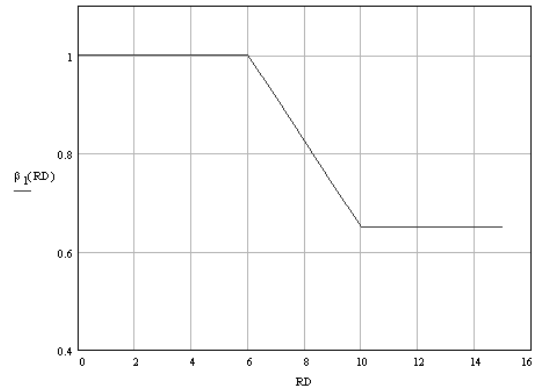


Figure 3: β_l for turbines in a cluster.

The Wake Added Turbulence

The total turbulence intensity is calculated from

$$I_{total} = \sqrt{I_{ambient}^2 + I_{park}^2} \quad (2)$$

References

[1] *Recommendation for the fulfillment of the requirements found in the technical criteria*, Danish Energy Agency, 1992

9. Turbulence Model – Frandsen & DIBt

S. Frandsen and M.L. Thøgersen [1] report an empirical turbulence model for calculating the integrated wake effect of turbines. This model takes into account the different structural fatigue responses of the structural materials considered, e.g. steel in the towers and hub extenders and glass fibre reinforced polyester (GRP) or glass fibre reinforced epoxy (GRE) in the blades. The equations below assume that the wind direction is approximately uniform distributed. Reference is made to Frandsen & Thøgersen [1] and Guidelines for the Design of Wind Turbines [2].

This model is included as a recommended model in the German DIBt Richtlinie [3].

Determining the Total Turbulence Intensity

The total turbulence intensity is determined from:

$$I_{T,total} = \left[(1 - N \cdot p_w) I_T^m + p_w \sum_{i=1}^N I_{T,w}^m(s_i) \right]^{1/m} \quad (1)$$

$$I_{T,w} = \sqrt{\frac{1}{[1.5 + 0.3 \cdot s_i \cdot \sqrt{v}]^2} + I_T^2} \quad (2)$$

where $p_w = 0.06$ (probability of wake condition)

$s_i = x_i / RD$

N is the number of closest neighboring wind turbines

m is the Wöhler curve exponent of the considered material

v is the free flow mean wind speed at hub height

x_i is the distance to the i -th turbine

RD is the rotor diameter

I_T is the ambient turbulence intensity (free flow)

$I_{T,w}$ is the maximum turbulence intensity at hub height in the center of the wake

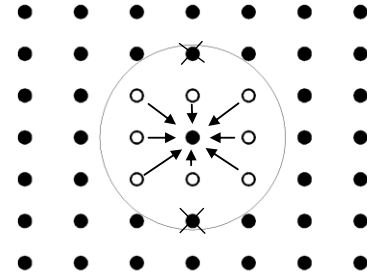
The number of closest neighboring turbines is determined as follows
– see also the figure to the right.

$N=1$: 2 wind turbines

$N=2$: 1 row

$N=5$: 2 rows

$N=8$: Wind farms with more than two rows



Increased Turbulence in Very Large Wind Farms

If the wind farm has more than five rows, the wind farm itself heavily influences the ambient wind climate. Also – if the distance between turbines in rows perpendicular to the predominant wind direction is less than 3 times the rotor diameter, an increase in mean turbulence level must be taken into account. This is done by substituting the ambient turbulence levels in (1) and (2) with the turbulence calculated from the equations (3) and (4).

$$I_T^* = 0.50 \cdot \sqrt{I_w^2 + I_T^2} + I_T \quad (3)$$

$$I_w = \frac{0.36}{1 + 0.08 \sqrt{s_r s_f} v} \quad (4)$$

where $s_r = x_r / RD$
 $s_f = x_f / RD$
 s_r is the distance *within* the row
 s_f is the distance *between* rows

References

- [1] S. Frandsen & M.L. Thøgersen, *Integrated Fatigue Loading for Wind Turbines in Wind Farms by Combining Ambient Turbulence and Wakes*, Wind Engineering, Volume 23, No. 6, 1999.
- [2] *Guidelines for Design of Wind Turbines*, DNV/Risø, Second edition.
- [3] Deutsches Institut für Bautechnik – DIBt, *Richtlinie für Windenergieanlagen, Einwirkungen und Standsicherheitsnachweise für Turm und Gründung*, Fassung März 2004.
- [4] S.T.Frandsen, *Turbulence and turbulence generated structural loading in wind turbine clusters*, Risø National Laboratory, January 2007.

10. Turbulence Model – D.C. Quarton & TNO Laboratory

A simple equation to determine the wake added turbulence has been proposed by D.C. Quarton and J.F. Ainslie [1]. The parameters in the equation have been re-calibrated by Quarton and Ainslie (the modified values) and also the Dutch TNO laboratory [2].

The main form of the equation is

$$I_{add} = K_I \cdot C_T^{\alpha_1} \cdot I_{amb}^{\alpha_2} \cdot (X / X_n)^{\alpha_3} \quad (1)$$

where K_I is a proportionality constant
 $\alpha_1, \alpha_2, \alpha_3$ are exponents
 X is the downstream distance (in meters)
 X_n is a characteristic wake length (either denoted near wake or far wake)
 I_{amb} is the ambient turbulence

The near wake length (X_n) is determined as described in the chapter dealing with the eddy viscosity wake model. In case of the TNO model, then the near wake length is replaced with a slightly different expression for the far wake length, see [2].

The proportionally constant and exponents are determined from the table below

Reference	K_I -Constant	α_1 -exponent	α_2 -exponent	α_3 -exponent
Quarton and Ainslie (original)	4.800	0.700	0.680	-0.570
Quarton and Ainslie (modified)	5.700	0.700	0.680	-0.960
Dutch TNO laboratory	1.310	0.700	0.680	-0.960

Note, that the ambient turbulence must be entered in percent (i.e. 10) when using the Quarton-Ainslie constants, while the TNO-constants are with ambient turbulence as decimal number (i.e. 0.10).

At the Figure 1 it is also easily seen, that the two models (Ainslie (modified) and TNO) actually are the same.

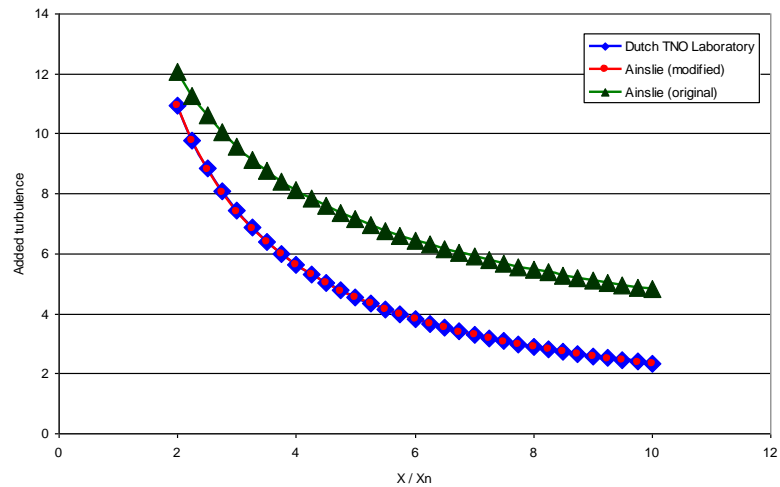


Figure 1: Wake Added Turbulence from the Three Models.

References

- [1] Quarton and Ainslie, *Turbulence in Wind Turbine Wakes*, Wind Engineering Vol 14 No 1
- [2] *European Wind Turbine Standards II*, ECN-C--98-096, December 1998.

11. Turbulence Model – B. Lange

The B. Lange turbulence model can only be used with the eddy viscosity wake model, because the turbulence parameters are derived directly from the eddy viscosity.

Turbulence within the Wake

The turbulence intensity, I_T , is defined as the standard deviation of the wind speed process divided by the mean wind speed, i.e.

$$I_T = \sigma_u / u_0$$

It is possible to relate the eddy viscosity to the turbulence intensity. According to Lange et al [1], the turbulence intensity within the wake can be calculated using the following relation below. Please note that the equation relates to the free wind speed, U_0 :

$$I_T = \varepsilon \frac{2.4}{\kappa \cdot U_0 \cdot z_h}$$

Alternative Empirical Approach

Another alternative empirical characterization of the wake turbulence was proposed by Quarton and Ainslie [2]. Their equation is based on a parameterization on the near wake length – which is primarily used in relation with the Eddy Viscosity model. They report, that the empirical turbulence decay is somewhat higher than other model predictions. The equation is:

$$I_{add} = 4.8 C_T^{0.7} I_{amb}^{0.68} [X / X_n]^{-0.57}$$

where I_{add} is the added turbulence intensity from the wind turbine wake
 I_{amb} is the ambient wind speed
 X is the downstream distance
 X_n is the near wake length

This alternative approach can also be used with other wake modes, as the near wake length is easily determined through empirical equations. For further details on the near wake length – please see the chapter on the Eddy viscosity wake model.

References

- [1] Lange, Bernard; H.P. Waldl; A.G. Guerrero; D. Heinemann & R.J. Barthelmie: *Modelling of Offshore Wind Turbine Wakes with the Wind Farm Program FLaP*, Wind Energy, 2003 6:87-104.
- [2] Quarton & Ainslie: *Turbulence in Wind Turbine Wakes*, Wind Engineering, Volume 14, No. 1.

12. Turbulence Model – G.C. Larsen

The G.C. Larsen is a simple empirical equation to determine the turbulence level within the wake. Reference is made to the paper ‘*Wind Field in Wakes*’ [1] and the European Research Project – European Wind Turbine Standards - EWTS II [2].

Turbulence Intensity

At positions downstream of the turbine, the wake added turbulence intensity can be determined from the equation:

$$I_w = 0.29 S^{-1/3} \sqrt{1 - \sqrt{1 - C_T}} \quad (1)$$

where S is spacing expressed in rotor diameters
 C_T is the thrust coefficient

The expression for turbulence intensity is only valid for distances larger than two rotor diameters downstream.

References

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12 Blockage

Introduction

Reduction in the AEP caused by upstream reduction from wind turbines in a wind farm is commonly referred to as blockage or global blockage. At the moment (2021) no industry wide consensus is reached on this topic.

Reduction of wind speeds upstream of a wind farm have been reported from mast pairs [1] and large variations in the upstream row of offshore wind farms have been reported in [2]. Large reduction of wind farm production in the order of 5% on the AEP have been shown from LES models and a simplified three layer model of the atmosphere [3]–[5]. The reduction of AEP in these models is due to interaction of the wind farm with the atmospheric boundary layer (ABL). For implementation in windPRO faster and more simplified models need to be chosen.

Blockage implementation in windPRO

For implementation in windPRO engineering type blockage models are used. In these models the thrust of the rotor is influencing the flow upstream of the wind turbines causing a deceleration in the wind speed. The induction will also cause potential speed ups on the side of the wind turbines that compensate some reduction in wind speed upstream of the wind turbines [6]. Within the windPRO implementation, blockage effects are only considered upstream of the wind turbine. Thus, blockage effects are only considered as a loss.

Solving the blockage equations is similar to calculating engineering wake models but in reversed order. As blockage and wake are both influencing the wake, in principle the model needs to be calculated iteratively. But since the effect of blockage is much smaller than for wakes convergence is usually reached after one iteration [2], blockage and wakes are calculated sequentially. This improves the calculation speed, as the models do not need to be calculated iteratively.

In to following two induction models are presented that are used as a basis for the blockage calculations in windPRO.

Self similar model

The self-similar model for the induction zone was developed by Forsting [7], [8]. It assumes self-similarity of the wind speed reduction in the induction zone. The model uses the following non-dimensional variables:

$$\tilde{u}(x, r) = \frac{u(x, r)}{u_\infty}, \epsilon = \frac{r}{r_1(x)}, \tilde{r}_{1/2}(x) = \frac{r_{1/2}(x)}{R}, \tilde{x} = \frac{x}{R}, \tilde{r} = \frac{r}{R}$$

$\tilde{r}_{1/2}$ is the radius where the wind speed reduction is half of the centre reduction. The wind speed in the induction zone consists of two terms, each dependant on the axial distance and radial distance respectively as seen in Equation 1:

$$\tilde{u}(\tilde{x}, \tilde{r}, C_T) = 1 - \alpha(\tilde{x}, C_T)f(\epsilon) \quad (1)$$

In axial direction the equation is as seen in Equation 1:

$$\alpha(\tilde{x}, C_T) = \frac{1}{2}(1 - \sqrt{1 - \gamma C_T}) \left[1 + \frac{\tilde{x}}{\sqrt{1 + \tilde{x}^2}} \right] \quad (2)$$

Only depending on the C_T of the turbine, the distance and a constant. In radial direction the following equations are needed, Equation 3 and 4:

$$\tilde{r}_{\frac{1}{2}}(\tilde{x}) = \sqrt{\lambda(\eta + \tilde{x}^2)} \quad (3)$$

$$f(\epsilon) = \text{sech}^a(\beta\epsilon) \quad (4)$$

The model is fitted with results from multiple CFD runs for several wind turbines. The modelling parameters are found as seen in Table 1:

Table 1. Parameter values fitted based on multiple CFD runs.

Constants	
γ	1.1
α	$\frac{8}{9}$
β	$\sqrt{2}$
λ	0.587
η	1.32

Vortex model

The vortex model for the induction zone is taken from Branlard [9]. The upstream influence of a turbine is described by a vorticity cylinder of the strength γ_t . We use Eq. 15 from [6] as the authors suggest – please see Equation 5:

$$\gamma_t = -2U_0[0.169C_T + 0.400C_T^2 - 0.482C_T^3 + 0.396C_T^4] \quad (5)$$

The model induced velocity in the induction zone is described by $u_{z,t}(r, z)$ in Equation 6.

$$u_{z,t}(r, z) = \frac{\gamma_t}{2} \left[\frac{R - r + |R - r|}{2|R - r|} + \frac{zk(z)}{2\pi\sqrt{rR}} \left(K(k^2(z)) + \frac{R - r}{R + r} \Pi(k^2(0), k^2(z)) \right) \right] \quad (6)$$

where:

$$k^2(z) = m(z) = \frac{4rR}{(R + r)^2 + z^2}$$

Where K is the complete elliptical integral of the first kind and Π the complete elliptical integral of the third kind. The elliptical integrals are computationally heavy to solve. For the far-field [10] present a solution (also known as the vortex dipole – see Equation 7) that avoids calculating these integrals.

$$u_x(r, x) = \frac{\gamma_t R^2}{4 r^2} \left[\frac{x^3}{(x^2 + r^2)^{\frac{3}{2}}} - \frac{x}{(x^2 + r^2)^{\frac{1}{2}}} \right] \quad (7)$$

Equation 7 is used from six rotor diameters onwards.

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